

Simple regression formulae

$$b_1 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

$$= \frac{\sum X_i Y_i - n \bar{X} \bar{Y}}{\sum (X_i - \bar{X})^2}$$

$$b_0 = \bar{Y} - b_1 \bar{X}$$

$$\text{Var}(b_1) = \frac{\sigma^2}{\sum (X_i - \bar{X})^2}$$

$$\text{Var}(b_0) = \sigma^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\sum (X_i - \bar{X})^2} \right)$$

$$\text{Cov}(b_0, b_1) = -\frac{\sigma^2 \bar{X}}{\sum (X_i - \bar{X})^2}$$

$$\text{SSTO} = \sum (Y_i - \bar{Y})^2$$

$$\text{SSE} = \sum (Y_i - \hat{Y}_i)^2$$

$$\text{SSR} = b_1^2 \sum (X_i - \bar{X})^2 = \sum (\hat{Y}_i - \bar{Y})^2$$

$$\sigma^2\{\hat{Y}_h\} = \text{Var}(\hat{Y}_h)$$

$$= \sigma^2 \left(\frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right)$$

$$\sigma^2\{\text{pred}\} = \text{Var}(Y_h - \hat{Y}_h)$$

$$= \sigma^2 \left(1 + \frac{1}{n} + \frac{(X_h - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right)$$

$$\hat{X}_h \pm \frac{t_{n-2, 1-\alpha/2}}{|b_1|} * \text{appropriate s.e.}$$

(valid approximation if $\frac{t^2 s^2}{b_1^2 \sum (X_i - \bar{X})^2}$ is small)

Working-Hotelling coefficient:

$$W = \sqrt{2F_{2, n-2; 1-\alpha}}$$

$$r = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum (X_i - \bar{X})^2 \sum (Y_i - \bar{Y})^2}}$$

$$S_{XX} = \sum_{i=1}^n (X_i - \bar{X})^2 = \sum_{i=1}^n X_i^2 - n\bar{X}^2$$

Regression in matrix terms

$$\text{Var}(\mathbf{Y}) = \text{E}[(\mathbf{Y} - \text{E}\mathbf{Y})(\mathbf{Y} - \text{E}\mathbf{Y})']$$

$$= \text{E}(\mathbf{Y}\mathbf{Y}') - (\text{E}\mathbf{Y})(\text{E}\mathbf{Y})'$$

$$\text{Var}(\mathbf{A}\mathbf{Y}) = \mathbf{A}\text{Var}(\mathbf{Y})\mathbf{A}'$$

$$\mathbf{b} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$$

$$\text{Var}(\mathbf{b}) = \sigma^2(\mathbf{X}'\mathbf{X})^{-1}$$

$$\hat{\mathbf{Y}} = \mathbf{X}\mathbf{b} = \mathbf{H}\mathbf{Y}$$

$$\mathbf{e} = \mathbf{Y} - \hat{\mathbf{Y}} = (\mathbf{I} - \mathbf{H})\mathbf{Y}$$

$$\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$$

$$\text{SSR} = \mathbf{Y}'(\mathbf{H} - \frac{1}{n}\mathbf{J})\mathbf{Y}$$

$$\text{SSE} = \mathbf{Y}'(\mathbf{I} - \mathbf{H})\mathbf{Y}$$

$$\text{SSTO} = \mathbf{Y}'(\mathbf{I} - \frac{1}{n}\mathbf{J})\mathbf{Y}$$

$$\sigma^2\{\hat{Y}_h\} = \text{Var}(\hat{Y}_h)$$

$$= \sigma^2 \mathbf{X}'_h (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}_h$$

$$\sigma^2\{\text{pred}\} = \text{Var}(Y_h - \hat{Y}_h)$$

$$= \sigma^2 (1 + \mathbf{X}'_h (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}_h)$$

$$R^2_{\text{adj}} = 1 - (n-1) \frac{MSE}{SSTO}$$