STA 302 / 1001 (A. Gibbs)
Additional Practice Problems for Chapter 2 of Sheather

1. What's wrong with the following simple linear regression model?

$$\mathrm{E}(Y_i|X = x_i) = \beta_0 + \beta_1 x_i + e_i$$

2. (a) For the simple linear regression model, what is the implication if $\beta_0 = 0$ so that the model is $Y_i = \beta_1 x_i + e_i$?
   (b) Derive the least squares estimator of β_1 for the model $Y_i = \beta_1 x_i + e_i$.
   (c) For a simple linear regression model, what is the implication if $\beta_1 = 0$ so that the model is $Y_i = \beta_0 + e_i$?
   (d) Derive the least squares estimator of β_0 for the model $Y_i = \beta_0 + e_i$ and show that it is unbiased.

3. Show:
   (a) $\sum_{i=1}^n \hat{e}_i x_i = 0$
   (b) $\sum_{i=1}^n \hat{e}_i \hat{y}_i = 0$

4. Consider a simple linear regression model. Assume all of the standard assumptions hold. And suppose that $\beta_0 = 10$, $\beta_1 = 5$, and $\sigma^2 = 4$.
   (a) What is the conditional distribution of $Y|X = x$ when $x = 0$? when $x = 5$?
   (b) When $x = 2$, what is the conditional probability that Y is between 16 and 20?

5. (Source: Exercise 1.11 in Kutner *et al.*)
   The regression function relating production output by an employee after taking a training program (Y) to the production output before the training program (X) is $\mathrm{E}(Y|X = x) = 20 + 0.95x$, where x ranges from 40 to 100. An observer concludes that the training program does not raise production output on the average because β_1 is not greater than 1.0. Comment.

6. (Source: Exercise 2.3 in Kutner *et al.*)
   A member of a student team playing an interactive marketing game received the following computer output when studying the relation between advertising expenditures (x) and sales (y) for one of the team's products:

   Estimated regression equation: $\hat{y} = 350.7 - 0.18x$
   Two-sided p-value for estimated slope: 0.91

   The student stated: "The message I get here is that the more we spend on advertising this product, the fewer units we sell!" Comment.