

STA 302/1001

Reminder Arriving in PM 107/108  
today 11-1 Friday 1-3  
to help with Assignment 3

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Now course: STA 365: Applied Bayesian Statistics  
in January

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unambiguous, accurate, deep

# Model from Example

- $Y$  - avg. # of flowers/plant  
(150, 300, 450, 600, 750, 900 plants/m<sup>2</sup>/s)
- 6 light intensities
  - 2 timings (early, late)

Model:  $Y = \beta_0 + \beta_1 i_{150} + \beta_2 i_{300} + \beta_3 i_{450} + \beta_4 i_{600} + \beta_5 i_{750}$   
 $+ \beta_6 \text{timing} + e$

all explanatory variables are indicator variables

Is timing important?  $H_0: \beta_6 = 0$  versus  $H_a: \beta_6 \neq 0$

test stat = 4.43

p-value calculated from  $t_{17}$  distribution  
 $= .0004$

After accounting for effect of intensity, strong evidence that mean of # of flowers/plot differs between early and late timing  
Holding intensity constant, get, on average, 12.16 flowers/plot more with early timing

What if we tested:  $H_0: \beta_1 = 0$  vs  $H_a: \beta_1 \neq 0$   
Strong evidence ( $p < .0001$ ) that  $\beta_1 \neq 0$  gives other variables in model

If  $\beta_1 = 0$ , model would be the same for intensities of 150 and 900

So we conclude that intensity of 150 gives, on average, a different  $\mu$  of flowers/plant than intensity of 900.

Individual tests for  $\beta_1, \dots, \beta_5$  compare mean of response at corresponding intensity to intensity of 900.

Now question: Does light intensity matter?

Want to test:  $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

vs  $H_a: \text{at least one of } \beta_1, \dots, \beta_5 \text{ is not } 0$

PARIA F-TEST - test whether a subset of  $\beta$ 's are 0  
simultaneously

Approach: ① fit the model with all predictor variables

"full model"

② fit the model without the predictor variables whose coefficients we're testing  
"reduced model"

$RSS$  in reduced model  $\geq$   $RSS$  in full model

$SS_{Reg}$  in reduced model  $\leq$   $SS_{Reg}$  in full model

$SS_T$  in reduced model =  $SS_T$  in full model

Test statistic

$$F_{obs} = \frac{(RSS_{red} - RSS_{full}) / (df_{error, red} - df_{error, full})}{RSS_{full} / df_{error, full}}$$

$df_{error, red} - df_{error, full} = \#$  of parameters that you are testing if all = 0

If can be shown that  $F_{obs}$  has an  $F$  distribution with  $(df_{error, red} - df_{error, full}, df_{error})$  degrees of freedom

Intuition: Did  $RSS$  go down by a statistically

→ significant amount when new predictors were added to model?

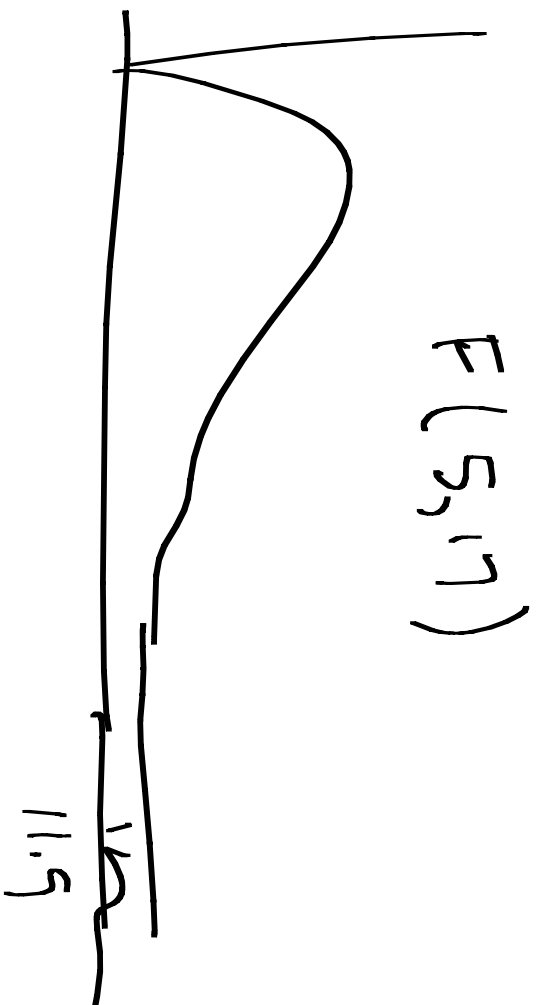
Equivalent to: Did  $R^2$  go up by a stat. sig. amount?

Back to our example:  $H_0: \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$   
 $H_a$ : at least 1 not 0

$$\text{Test stat: } F_{obs} = \frac{3450.99 - 767.47}{5} = 11.9$$

$\frac{767.47}{17}$

P-value



P-value will  
be very  
small

So strong evidence that not all of  $\beta_1, \dots, \beta_5$  are 0  
So there is an intensity effect  
The intensity is in the words

This is decomposing SS Reg into 2 components  
Intensity  
Turnint



Could write ANOVA table to report this

Source	df	SS	MS	F
Regression	1	886.95	$\frac{886.95}{1}$	$\frac{886.95}{45.15}$
Error (intensity)	5	2683.52	$\frac{2683.52}{5}$	$\frac{537.70}{45.15} = 11.9$
Error	17	$\frac{767.47}{17}$	45.15	
Total	23	4337.94		

Note  $886.95/45.15 \sim 19 = (4.43)^2$

Note: could carry out a partial F-test to test  $H_0: \beta_j = 0$   
 vs  $H_a: \beta_j \neq 0$

(assuming all other variables are in model) (i.e. one parameter)  
F test stat is square of t-test stat.