Exercises 1 STA 3000, 2020

1. Consider the decision problem given by $\mathcal{X} = \{1, 2, 3, 4\}, \Theta = \{a, b\}, \Psi(\theta) = \theta, L(\theta, \psi) = 1 - I_{\{0\}}(\theta - \psi)$ and the model is given by the following table.

	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$\theta = a$	$1/4$	$1/4$	0	$1/2$
$\theta = b$	$1/2$	0	$1/4$	$1/4$

(a) Calculate the risk function of the decision function given by $d(1) = a, d(2) = a, d(3) = a$ and $d(4) = b$.
(b) Is d admissible?
(c) Calculate the prior risk of d when the prior is given by $\pi(a) = 1/4, \pi(b) = 3/4$.
(d) Obtain a Bayes rule.
(e) Calculate the risk function and the prior risk of the decision function δ given by

	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$\delta(\cdot, \{a\})$	$1/2$	1	$1/3$	$2/3$
$\delta(\cdot, \{b\})$	$1/2$	0	$2/3$	$1/3$

(f) From the point-of-view of frequentist risk which decision function is best among those considered in this question?

2. Suppose that X takes values in $\{\theta - 1, \theta + 1\}$ with equal probabilities where θ is unknown in $\mathbb{Z}$ (the integers). After observing X the decision problem is to estimate θ and we use the loss function $L(\theta, a) = \min\{|\theta - a|, 1\}$.
(a) Calculate the risk function of the nonrandomized decision function $d(X) = X + 1$ if $X < 0$ and $d(X) = X - 1$ if $X \geq 0$.
(b) Calculate the risk function of the randomized decision δ_p which selects $X - 1$ or $X + 1$ with probabilities p and q respectively.
(c) Which of δ_p and d is preferred?

3. Suppose that the conditional density of X given θ is $\exp(-|x - \theta|/2)$ where θ has prior density $\exp(-|\theta - \eta|/2)$ on R^1 for some fixed η. Further suppose that $\Psi(\theta) = \theta$ and the loss function is $L(\theta, a) = (\theta - a)^2$. Find a Bayes rule based on observing x.