

1. Suppose  $x \in \mathcal{X} = \{1, 2, 3\}$  and we have the following statistical model.

$\theta \backslash x$	1	2	3
a	$1/2$	$1/3$	$1/6$
b	$1/4$	$1/4$	$1/2$
c	$1/5$	$4/5$	0
d	$1/3$	$1/2$	$1/6$

Suppose further that we are interested in the parameter  $\tau = \mathbb{E}(x)$  where  $\mathbb{P}(a) = \mathbb{P}(b) = 1$  and  $\mathbb{P}(c) = \mathbb{P}(d) = 2$ .

(a) Determine all the possible profile likelihood functions (equivalent up to positive constant multiples).

(b) For each of profile likelihoods determine a .5-profile likelihood region for  $\tau$ .

2. Consider a sample of  $n$  from the  $N(\mu, \sigma^2)$  model with  $(\mu, \sigma^2) \in (\mathbb{R} \setminus \{0\}) \times (0, \infty)$ .

(a) Write the likelihood function in terms of the parameter  $(\tau, \sigma^2)$  where  $\tau = \sigma/\mu$ .

(b) Determine the profile likelihood for  $\tau$ .