5.1.10 We get an approximate value of $P(C)$ by dividing the number of sample values lying in the set C by the sample size, i.e., $P(C)$ is determined by $\bar{I}_C = n^{-1}\sum_{i=1}^{n} I_C(X_i)$ for a sample $X_1, \ldots, X_n$. The weak law of large numbers (see Theorem 4.2.1) guarantees that $\bar{I}_C$ is close to $P(C)$ when the sample size is big. However, the accuracy depends on the size of $P(C)$. Consider the central limit theorem (see Theorem 4.4.3), $(\bar{I}_C - P(C))/(P(C)(1-P(C))/n)^{1/2} \xrightarrow{D} N(0,1)$. When $P(C)$ is very close to 0 or 1, a small change of $\bar{I}_C$ could lead to a large difference from the value $P(C)$.

5.2.17 The conditional probability $P(X = x|X > 5) = \theta(1-\theta)^{x-6}$ where $x \geq 6$ and $\theta = 1/3$. The conditional probability function is decreasing and the value $x = 6$ is the most probable.
Again the shortest interval containing 95% probability of a future X is $[6, c]$ satisfying $P(6 \leq X \leq c|X > 5) \geq 0.95$. Since $P(X \leq x|X > 5) = 1-(1-\theta)^{x-5}$, the solution is $c \geq 5 + \ln(0.05)/\ln(2/3) = 12.3884$. Finally, the interval $[6, 13]$ is the solution.

5.3.15 The first quartile c, of a $N(\mu, \sigma^2)$ distribution satisfies

$$0.25 = \int_{-\infty}^{c} \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\} dx = \Phi\left(\frac{c-\mu}{\sigma}\right).$$

Therefore, $c = \mu + \sigma z_{.25}$, where $z_{.25}$ is the first quartile of the $N(0,1)$ distribution, i.e., $\Phi(z_{.25}) = .25$. But we see from this that several different values of (μ, σ^2) can give the same first quartile, e.g., $(\mu, \sigma^2) = (0, 1)$ and $(\mu, \sigma^2) = (z_{.25}/2, 1/4)$ both give rise to normal distributions whose first quartile equals $z_{.25}$. Therefore, we cannot parameterize this model by the first quartile.

5.3.18 If P_1 is the true probability measure, the sample mean $\bar{X} = (X_1 + \cdots + X_n)/n$ has a $N(1, 1/100)$ distribution. And $\bar{X}$ has a $N(0, 1/100)$ distribution if P_2 is true. Hence, we conclude the true probability measure is P_1 if $\bar{X} \geq 1/2$ and is P_2 if $\bar{X} < 1/2$. The probability of making an error is $P_1(\bar{X} < 1/2) = P_1((\bar{X}-1)/\sqrt{1/100} < (1/2-1)/\sqrt{1/100}) = \Phi(-5) = 2.8665 \times 10^{-7}$. Thus, this inference is very reliable.