Assignment 1
STAC70S, 2012

1. Prove that if $X_1, \dots, X_k$ are random variables defined on Ω and $B_1, \dots, B_k \subset R^1$ then $(X_1, \dots, X_k)^{-1}(B_1 \times \cdots \times B_k) = X_1^{-1}B_1 \cap X_2^{-1}B_2 \cap \cdots \cap X_k^{-1}B_k$.

2. Prove that if $\mathcal{A}$ is a σ-field on a set Ω, then
(a) $\phi \in \mathcal{A}$,
(b) if $A_1, A_2, \dots, A_n \in \mathcal{A}$ then $\cup_{i=1}^{n} A_i \in \mathcal{A}$ and $\cap_{i=1}^{n} A_i \in \mathcal{A}$,
(c) if $A_1, A_2, \dots \in \mathcal{A}$ then $\cap_{i=1}^{\infty} A_i \in \mathcal{A}$.

3. Prove that if $\{\mathcal{A}_i : i \in I\}$ is a collection of σ-fields on a set Ω, then $\cap_{i \in I} \mathcal{A}_i$ is a σ-field on Ω.

4. If $X \sim$ log-$N(\mu, \sigma^2)$ then prove that (i) $E(X) = \exp\{\mu + \sigma^2/2\}$, (ii) $Var(X) = \exp\{2\mu + \sigma^2\}(\exp\{\sigma^2\} - 1)$.

5. If $X \sim t_1$, then prove that $E(X)$ does not exist.

6. (i) If a stochastic process $\{X_t : t \in T\}$, with $T \subset R^1, 0 \in T, X_0 = 0$ has independent increments and $Var(X_t) = \sigma^2 t$ for all t, then $Cov(X_t, X_s) = \sigma^2 \min\{s, t\}$.
(ii) Prove that a random walk has independent increments.

7. Prove that the Brownian bridge is a Gaussian process.

8. Text Exercise 2.10.