Problems for Week #2

For each of the questions $p \in (0,1)$, $\lambda > 0$, $q = 1-p$, m are constants.

#1. Verify that each of the following are probability functions.

(a) $f(x) = p^x q^{1-x}$, $x = 0, 1$
$\quad = 0$, otherwise (ow)

(b) $f(x) = \binom{m}{x} p^x q^{m-x}$, $x = 0, 1, \cdots, m$
$\quad = 0$, ow

(c) $f(x) = q^{x-1} p$, $x = 1, 2, \cdots$
$\quad = 0$, ow

(d) $f(x) = e^{-\lambda} \frac{\lambda^x}{x!}$, $x = 0, 1, \cdots$
$\quad = 0$, ow

#2. Calculate the mean and the variance for each of the distributions in # 1.

#3(a) The probability generation function G is defined for rv's have ranges which are $\subset \{0, 1, 2, \cdots\}$.

If X is such a rv we will call it a <u>counting rv</u>. Its pgf is

$$G(s) = E(s^X),$$

for those s's such that $E(|s|^X) < \infty$.

For each pf in #1 calculate the corresponding pgf (probability generating function).

(b) The moment generating function (mgf) of X is defined as

$$m(t) = E(e^{tX}),$$

for those t's for which $E(e^{tX}) < \infty$. Calculate the mgf for each pf in #1.

4(a) Show $X = 0 \Rightarrow E(X) = 0$.

(b) Show $X \le Y \Rightarrow E(X) \le E(Y)$

(c) Show $|E(X)| \le E(|X|)$

(d) Show $E(|X+Y|) \le E(|X|) + E(|Y|)$

5 (challenge) Let $0 \le X_1 \le X_2 \le \cdots$ & suppose $\lim_{n \to \infty} X_n(s) = X(s)$, $\forall s \in S$. Show $E(X_n) \to E(X)$.