Test **STA257** **Time**:3hrs

Instructions: The test is out of 100 and each question is worth 7. Your maximum grade is 100. See the end for some useful information. Please, at most 1 question/page in your booklets. No aids allowed.

1. Let Y be $binomial(15, 1/3)$. Evaluate $Var(Y)$. Note: You must show your work.
2. Let U be a $uniform((0,1))$ rv. Set $Y = -log(U)$. Calculate the pdf of Y. Now suppose Z is $Bernoulli(1/3)$. Find and sketch a function $g : (0,1) \to \mathbb{R}$ such that $Z \stackrel{d}{=} g(U)$. Hint: consider the function $g(u) = glb\{z : F(z) \geq u\}$, $0 < u < 1$. Here F is the df of Z.
3. .Two people each roll a fair 6 sided die. Let X be the number thrown by the first person and Y the number thrown by the second. You may assume these are independent. Let $V = |X - Y|$ and $M = max\{X, Y, \}$. Calculate $E(V)$ and $E(M)$.
4. Show $E(|X|) = 0$ implies $P(X = 0) = 1$.
5. Let A and B be independent events. Show that A and B^c are also independent.
6. Let $P(A) = P(B) = 1$. Show $P(AB) = 1$.
7. Let X, Y and Z be independent Poisson rv's. Show that $X + Y + Z$ is Poisson.
8. Let $X \sim Poisson(3)$ be independent of $Y \sim Poisson(6)$. Set $W = X + Y$. Calculate $P(X = k|W = 3)$ for $k = 0, 1, 2, 3$.
9. Let $X \sim binomial(10, p)$ be independent of Y. If $X+Y \sim binomial(12, p)$ show $Y \sim binomial(2, p)$.
10. Let $Z_1, Z_2, \ldots$ be *iid* $Bernoulli(1/3)$ and let $S_n = Z_1 + \cdots + Z_n$. Let T denote the smallest n such that $S_n = 3$. Calculate $Var(T)$.
11. Toss a fair coin. If H obtains you select 2 chips with replacement from Hat#1. Otherwise you select 3 chips without replacement from Hat#2. Hat#1 contains 3 red chips and 4 black chips while Hat#2 contains 5 reds and 2 black chips. Let A={at least 1 red chip is selected}. Calculate $P(H|A)$.
12. Let $X \sim geometric(1/3)$. Calculate $P(X > 1)$ and $Var(X)$.
13. A rv X has pgf given by $G(s) = E(s^X) = .1s + .4s^4 + .5s^{16}$. Calculate $E(\sqrt{X})$.
14. Let $X \geq 0$ be either a counting or a continuous rv with df F. Assuming $E(X^2) < \infty$, show $E(X) = \int_0^\infty 1 - F(x)dx$.
15. Let X have mean μ and standard deviation σ. Use Markov's inequality to show $P(|X - \mu| \geq 2\sigma) \leq 1/4$.

Information

A $Bernoulli(p)$ rv can only take on 1 or 0 with probabilities p and $q = 1 - p$, respectively.

The $geometric(p)$ probabilities are $q^{k-1}p, k = 1, 2, ...$

$1 + x + x^2 + \cdots = 1/(1 - x) for |x| < 1$

The $Poisson(\lambda)$ probabilities are $e^{-\lambda}\lambda^k/k!$

The $multinomial(N; p_1, \ldots, p_k)$ probabilities are $\frac{N!}{(i_1!)\ldots(i_k!)} p_1^{i_1} \cdots p_k^{i_k}, i_1 + \cdots + i_k = N$. Here $p_1 + \cdots + p_k = 1$. $k = 2$ yields the binomial which may also be thought of as a sum of k *iid* $Bernoulli(p)$ rv's.

A $uniform((0, 1))$ rv has *pdf* $f(x) = 1$ for $0 < x < 1$ and is 0 otherwise.

The indicator rv of an event A is denoted by I_A or $I(A)$. This is a function from the sample space to rhe reals with range $\{0, 1\}$.

A sequence A_n, $n = 0, 1, \ldots$ is said to be increasing if $A_1 \subset A_2 \subset \cdots$ and is decreasing if $A_1 \supset A_2 \supset \cdots$.

We say $A_n \to A$ if $I(A_n) \to I(A)$. In the increasing case we write $A_n \uparrow A$. In the decreasing case we write $A_n \downarrow A$.