We solved:

- $|E(X)| \leq E|X|$
- $Y \geq 0$ & $0 < k_1 < k_2$ (integers) then $E(Y^{k_2}) < \infty \Rightarrow E(Y^{k_1}) < \infty$
- $\sum |X_k| < \infty$ & $\sum E(|X_k|) < \infty \Rightarrow E(\sum X_k) = \sum E(X_k)$

  (the sol'n used the DCT)
- X discrete with range $\{x_1, x_2, \ldots\}$ & $g: \mathbb{R} \to \mathbb{R}$ satisfying $\sum |g(x_k)| I(X = x_k) < \infty$ & $\sum |g(x_k)| P(X = x_k) < \infty$

$$\Rightarrow E[g(X)] = \sum g(x_k) P(X = x_k)$$

~~pf~~ $f(x) = P(X = x)$, $x \in \mathbb{R}$ & for discrete rv's

$$f(x) \geq 0, \quad \sum_{x \in \mathbb{R}} f(x) = 1$$

<u>Some discrete dist'ns:</u>

Bernoulli(p), binomial(N, p), geometric(p), negative binomial, Poisson(λ), hypergeometric

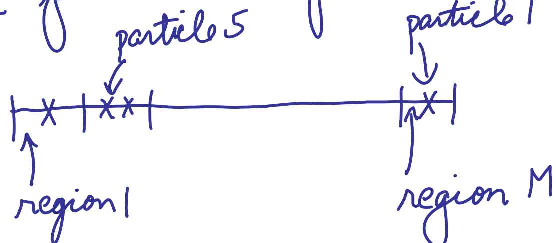
Please review these.

- pgf $G(z) = E(z^X)$ for counting rv's

$$= \sum_{k=0}^{\infty} z^k P(X = k)$$

- $|G(z)| \leq 1$ if $|z| \leq 1$.
- $G(z)$ is a power series & the interchange of $\frac{d^{()}}{dz^{()}}$ with $\sum_{k=0}^{\infty}$ follows easily when $|z| < 1$. It continues to hold at $z = 1$ but then LH derivatives are then used.

- def'n of independence of rv's & events
- an example of N iid uniform$(\{1, \cdots, M\})$ rv's
- N <u>distinct</u> particles placed in M regions

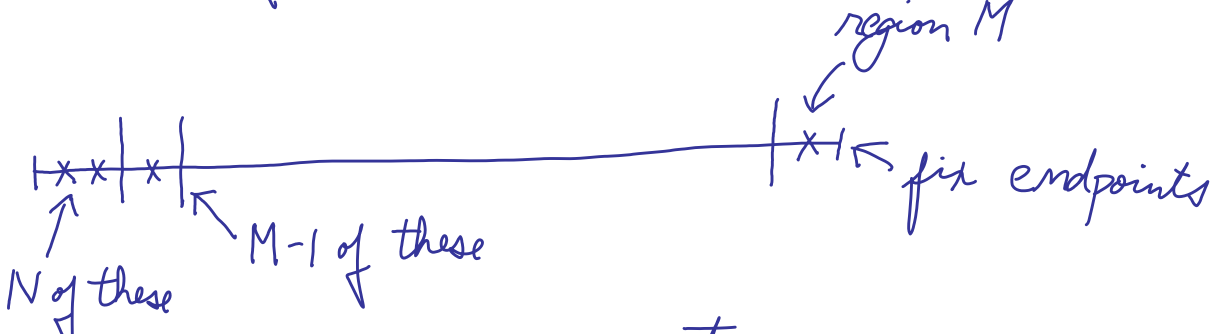


then the # of different arrangements is M^N
& if each arrangement is equally likely (ie $\frac{1}{M^N}$)
then the <u>counts</u> in the regions are
multinomial. So if N_i = # of particles in region i
then

$$P(N_1 = n_1, \cdots, N_M = n_m) = \binom{N}{n_1, \cdots, n_m} \underbrace{\left(\frac{1}{M}\right)^{n_1} \cdots \left(\frac{1}{M}\right)^{n_m}}_{M^N}$$

- if the N particles are <u>indistinguishable</u>
then the # of different arrangements is

$$\binom{N+M-1}{N}$$



Move x's + |'s to get all arrangements.

- pgf of a binomial (N, p) is
$$G(z) = (q + pz)^N$$
- pgf of a Poisson (λ) is
$$G(z) = e^{\lambda(z-1)}$$

Moments can be obtained by differentiating G at $z = 1$. So if X has pgf G then
$$G^{(1)}(0) = E(X), \quad G^{(2)}(0) = \underbrace{E[X(X-1)]}_{\text{a factorial moment}}, \text{ etc} \ldots$$

- μ & σ^2 can be obtained from $E(X)$ & $E[X(X-1)]$

- $X_1, \ldots, X_m$ independent
$$\Rightarrow G_{X_1 + \cdots + X_m}(z) = G_{X_1}(z) \times \cdots \times G_{X_m}(z)$$

In particular a finite sum of ind Poisson rv is is Poisson and a finite sum of ind binomial's (with the same p) is binomial.

Try N apples. 2 containers. Count the # of ways of placing the apples into the containers if (i) the apples are distinguishable & (ii) they aren't.