

- notion of a stationary time series
- Toeplitz matrices
- inverse in the Markov case
- Cauchy Schwartz
- multinomial, uniform
- dist'n of a count when placing  $N$  indistinguishable items into  $M$  cells/regions
- Poisson spacial process
- Poisson counting process
- distribution function, tail probability function
- exponential ( $\lambda$ ) distribution, gamma distribution
- conditional P, conditional E (basic)

### Related Sections in the text

4.2, 4.4 (especially vector p.f.'s), 4.7, 4.8, 5.1, 5.2, 5.3 (only in the discrete & its cases)

### Suggested problems

p 37 # 7

p 57 #'s 2, 5 (read)

p 60 #'s 2 (challenge), 5

p65 #1 4, 5, 8

p74 #1 1, 5 (read)

p78 #1 2 (use conditioning), 3, 4, 5

p83 #1 1, 2 (assume the exponential)

p86 #1 1, 3

p91 #1 1, 2 (read), 4

### Some Further Problems

Let  $X$  be a r.v. Set

$$F(x) = P(X \leq x), \quad \forall x$$

$$\bar{F}(x) = P(X > x), \quad \forall x$$

Problem 1 Show  $\bar{F}$  is decreasing, right cts.,  
 $\bar{F}(-\infty) = 1, \bar{F}(\infty) = 0$

Problem 2 Let  $X \sim \text{exponential}(\lambda)$ . Show

$$\bar{F}(s+t) = \bar{F}(s) \bar{F}(t), \quad \forall s, t \geq 0$$

+ hence  $P(X > s+t | X > s) = P(X > t)$

(This is the ageless or memoryless property.)

Problem 3 Let  $\{N(t) : t \geq 0\}$  be a Poisson counting process of rate  $\lambda$ . Let  $S_n =$  time to the  $n$ th point. Derive the pdf of  $S_n$ .

Problem 4 Let  $X_1, X_2, \dots$  be iid exponential(1). Place points at times  $S_n = X_1 + \dots + X_n, n=1, 2, \dots$ . Now let  $N(t) = \#$  of points in  $[0, t]$ . Show  $N(t) \sim \text{Poisson}(\lambda t)$

Problem 5 Let  $\{N(t) : t \geq 0\}$  be a Poisson process of rate  $\lambda$ . You are given that  $N(1) = m$ . Denote the times of those  $m$  points by  $T_1, \dots, T_m$ . Obtain the pdf for each of  $T_m$  and  $T_1$ .

Problem 6 Let  $Y_1, \dots, Y_m$  be iid uniform  $([0, 1])$ . Consider the order statistics  $Y_{(1)} < Y_{(2)} < \dots < Y_{(m)}$   
 $\{ Y_{(m)} = \max\{Y_1, \dots, Y_m\}, Y_{(1)} = \min\{Y_1, \dots, Y_m\} \}$

Derive the pdf for each of  $Y_{(1)}$  &  $Y_{(n)}$ .  
Compare with Problem 5.

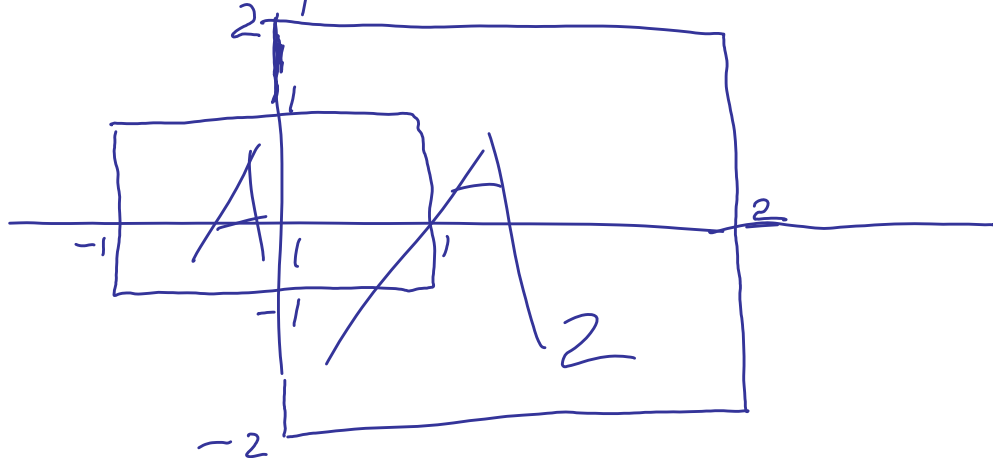
Problem 7 (Challenge) You have  $N$  towns with at most one road between any two towns. The total # of roads is  $M$ . Show that  $\exists$  a collection of towns  $\Rightarrow$  the # of roads leading into the collection from outside the collection is at least  $M/2$ . Hint: Go to each town & toss a fair coin.

Problem 8 Let  $X, Y$  be counting rv's. Show  
$$E(Y) = E[E(Y|X)]$$

Problem 9 Let  $X_1, X_2, \dots$  be iid exponential(1) &  $N \sim \text{Poisson}(10)$  ind of the  $X$ 's. Set  
$$S_N = X_1 + \dots + X_N ; S_0 = 0$$

Calculate  $E(S_N)$  and  $\text{Var}(S_N)$ .

Problem 10 Consider a Poisson process of rate 1 on  $\mathbb{R}^2$  & let  $A_1$  &  $A_2$  be as in the picture.



Calculate the joint pgf of  $N(A_1)$  &  $N(A_2)$ .

Problem 11 Let  $\{N(t) : t \geq 0\}$  be a

Poisson process of rate  $\lambda$ . Show

- $\{N(t) : t \geq 0\}$  has independent increments
- $P(N((t, t+h]) > 1) = o(h)$   $\left\{ \frac{o(h)}{h} \rightarrow 0 \right\}$
- $P(N((t, t+h]) = 1) = \lambda h + o(h)$

Problem 12  $X \sim \text{Poisson}(4)$  ind of  $Y$  (a counting rv)

Show  $X + Y \sim \text{Poisson}(10) \Rightarrow Y \sim \text{Poisson}(6)$