Week 7

Matching problem (2 sol'ns), "best prize" problem, the exponential & gamma dist'ns, the ageless or memoryless property, hazard / failure rate functions.

Readings from the text

Sections 4.6, 4.7, 4.8, 8.1, 8.2, 8.3

8.1, 8.2, 8.3: similar to STA257

we will discuss these further

Problems from the text

p76 # 5 (read)

p78 #'s 3, 4, 5

p83 #'s 1, 3, 4

p143 #'s 1, 2, 4

Further problems

1- Let $X_1, \ldots, X_n$ be iid with pdf f & df F.

Use the multinomial to obtain the joint pdf of $X_{(r_1)}$ & $X_{(r_2)}$ where $r_1 < r_2$. Extend this to $X_{(r_1)}, \ldots, X_{(r_k)}$ where $r_1 < \cdots < r_k$.

2- For 1 obtain the joint pdf of $X_{(1)}, \cdots, X_{(n)}$

3- A rv $X \geq 0$ is such that $P(X=0) \neq 1$ and

$$\bar{F}(s+t) = \bar{F}(s)\,\bar{F}(t) \quad , \quad s, t \geq 0 \quad ,$$

where $\bar{F}(x) = P(X > x)$.

(a) Show $0 \leq \bar{F} \leq 1$, $\bar{F}$ decreases, $\bar{F}(-\infty) = 1$ & $\bar{F}(\infty) = 0$, $\bar{F}$ is right cts (Note: (a) is for any rv X — you may use the properties of df's)

(b) Show $\bar{F}(0) = 1$ & $0 < \bar{F}(t) < 1$ for any $t > 0$

(c) Show $\bar{F}(n) = e^{-\lambda n}$, $n = 0, 1, \cdots$ for some $\lambda > 0$

(d) Show $\bar{F}\left(\frac{n}{m}\right) = e^{-\lambda\left(\frac{n}{m}\right)}$, $n, m = 1, 2, \cdots$

(e) Use (d) to conclude $\bar{F}(x) = e^{-\lambda x}$, $\forall\, x > 0$

(f) Obtain the pdf of X & the failure rate function.

4- For #1 obtain the df of $X_{(r)}$ without using the heuristic multinomial approach.

5- A Poisson process of rate λ on $t \geq 0$ has n points in $[0,1]$ at times $T_1 < \cdots < T_n$. Show $T_1, \cdots, T_n$ has the same dist'n as the order statistics for a sample of size n from a uniform $([0,1])$.