Some Sol'ns

#'s 1 + 2 done in class

#3 $E(|X|) = 0 \Rightarrow 0 \le |E(X)| \le E(|X|) = 0$

$$\Rightarrow |E(X)| = 0$$

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#4 Assume $X \stackrel{as}{=} 0 \Rightarrow |X| \stackrel{as}{=} 0$. Now

$$|X| = \underbrace{|X|\, I(|X| = 0)}_{0} + \sum_{k=0}^{\infty} |X|\, I(k < |X| \le k+1)$$

$$\Rightarrow E(|X|) = E(0) + \sum_{k=0}^{\infty} E\big(|X|\, I(k < |X| \le k+1)\big)$$

$$\le 0\, E(1) + \sum_{k=0}^{\infty} E(k+1)\, I(k < |X| \le k+1)$$

$$= 0 + \sum_{k=0}^{\infty} (k+1)\, P(k < |X| \le k+1)$$

But for $k \ge 0$

$$0 \le P(k < |X| \le k+1) \le P(|X| > 0) = 0$$

$\therefore$ $0 \le E(|X|) \le 0 \Rightarrow E(|X|) = 0$

$$\Rightarrow E(X) = 0$$

p31 #5 — in class

p31 #6 — Nothing to do but read the question.

p31 #7 — Apply the theorem or calculate

$$\begin{pmatrix} 1 & \beta & \beta^2 & \cdots & \beta^n \\ \beta & 1 & \beta & \ddots & \vdots \\ \beta^2 & \ddots & \ddots & \ddots & \beta^2 \\ \vdots & \ddots & & & \\ \beta^n & \cdots & \beta^2 & \beta & 1 \quad \beta \end{pmatrix}^{-1}$$

as a tri-diagonal matrix.

p43 #6, 8, 9 (trivial)

p45 #5

$$I\left(\bigcup_i A_i\right) \le \sum_i I(A_i)$$

$$\Rightarrow P\left(\bigcup_i A_i\right) \le \sum_i P(A_i)$$

(14) is similar.

p45 #7

$$X \in \{0, 1, \dots\} \Rightarrow X = \sum_{n=0}^{\infty} I(X > n)$$

$$\Rightarrow E(X) = \sum_{n=0}^{\infty} E(I(X > n)) = \sum_{n=0}^{\infty} P(X > n)$$