Assignment1 STA447

Note: Due Thursday, February17 at the beginning of the lecture. Please try to keep a copy of the original . This assignment is worth 16% of your final grade. Plagerism $\Rightarrow$ 0 . **No** late assignments .

1. Let $\{X_t : t=0,1,...\}$ be a Galton Watson branching process with $X_0=1$ and geometric(p) offspring distribution - $0<p<1$. Let T be the first time the population becomes extinct. Obtain $P(T=k)$ and determine which values of p lead to $E(T) < \infty$.

2. Let $\{F_t$, t in T$\}$ be σ-fields of events. Show $\bigcap_{t\in T} F_t$ is also a σ-field.

Give an example of 2 σ-fields F_1, F_2 where $F_1 \cup F_2$ is not a σ-field .

3. For $a < b$ in $\Re$ show $\sigma(\{(a,b)\}) = \sigma(\{[a,b]\}) = \sigma(\{\text{open subsets}\})$.

4. Show $X_n \xrightarrow{ms} X \Leftrightarrow X_n - X_m \xrightarrow{ms} 0$, as $n,m \to \infty$.

Hints: (a)Recall that p-convergent sequences have subsequences converging almost surely. (b) It is a direct consequence of the MCT and the definition of liminf that for $X_n \geq 0$ $E(\underline{\lim} X_n) \leq \underline{\lim} E(X_n)$. This is called Fatou's Lemma .

5. (a)Show $X_n \xrightarrow{p} 0 \Leftrightarrow E\left(\frac{|X_n|}{1+|X_n|}\right) \to 0$.

(b) Does there exist a sequence of independent rv's X_1, X_2 ,... such that $P(\sum_{k=1}^{\infty} |X_k| < \infty) = 1/2$?

6. For X, Y in L_2 define $\langle X,Y\rangle = E(XY)$ and $||X|| = \sqrt{\langle X,X\rangle}$. Verify

(i) $\langle aX+bY,Z\rangle = a\langle X,Z\rangle + b\langle Y,Z\rangle$

(ii) $||X+Y||^2 + ||X-Y||^2 = 2||X||^2 + 2||Y||^2$

(iii) if $i \neq j \Rightarrow \langle X_i, X_j\rangle \neq 0$ then

$$||\sum_{i=1}^{n} X_i||^2 = \sum_{i=1}^{n} ||X_i||^2$$

7. Let $\{N(t), t \geq 0\}$ be a renewal process with iid interarrival times $X_i \geq 0$ with $E(X_i) = \infty$. Show $E(N(t))/t \to 0$ as $t \to \infty$.

8. Let $\{N(t), t \geq 0\}$ be a renewal process with iid interarrival times $X_i \geq 0$ having mean μ and (finite) variance σ^2. Show

$$\frac{N(t) - t/\mu}{\sqrt{t\sigma^2/\mu^3}} \xrightarrow{d} N(0,1), \text{ as } t \to \infty.$$