STA 2004F Efficiency of RB design compared to CR design

The efficiency of a randomized block design with r blocks and v units per block is estimated by

$$\frac{SS_B + r(v-1)MS_E}{(rv-1)MS_E} \tag{1}$$

where SS_B is the sum of squares due to blocks, and MS_E is the mean square error in the analysis of variance table for the randomized block design. This table is

Source	df	SS	MS	$E_R(MS)$
treatments	$v-1$	$\Sigma_{js}(\bar{y}_{j.} - \bar{y}_{..})^2$	MS_T	$\frac{r}{v-1}\Sigma_j(\bar{\xi}_{j.} - \bar{\xi}_{..})^2 + \frac{r}{v-1}\Sigma\tau_j^2$
blocks	$r-1$	$\Sigma_{js}(\bar{y}_{.s} - \bar{y}_{..})^2$	MS_B	$\frac{v}{r-1}\Sigma_s(\bar{\xi}_{.s} - \bar{\xi}_{..})^2$
error	$(v-1)(r-1)$	SS_E	MS_E	$\frac{1}{r(v-1)}\Sigma_{js}(\xi_{js} - \bar{\xi}_{.s})^2$

where ξ_{js} is the unit constant for the unit in the sth block that gets treatment T_j. The expected value of MS_E is derived in Hinkelmann and Kempthorne, p.256, and arises due to the restricted randomization used in the design.

Now in a CR design, the variance in the randomization distribution of the difference between two treatment means, $\bar{Y}_j - \bar{Y}_{j'}$, say, is $\frac{2}{r}\Sigma_{js}(\xi_{js} - \bar{\xi}_{..})^2/(rv-1)$. From the table above we see that $\Sigma_{js}(\xi_{js} - \bar{\xi}_{..})^2$ is estimated by $r(v-1)MS_E + (r-1)MS_B = r(v-1)MS_E + SS_B$. Thus the variance of the difference can be estimated by

$$\frac{2}{r}\left\{\frac{r(v-1)MS_E + SS_B}{rv-1}\right\}.$$

In the RB design $\text{var}(\bar{Y}_j - \bar{Y}_{j'})$ is estimated by $\frac{2}{r}MS_E$. The ratio of these two numbers is the efficiency expression (1).