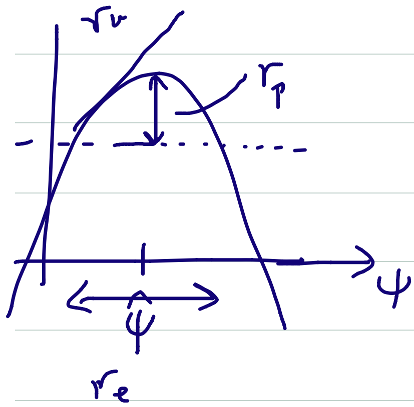
$\theta = (\psi, \lambda) \qquad \psi \in \mathbb{R}$

3 approx$^{\underline{n}}$s : $\pm\sqrt{2\{\ell_p(\hat\psi)-\ell_p(\psi)\}} = r_p(\psi)$

$\dot\sim\ N(0,1)$ under $f(y;\theta)$



$$\boxed{\begin{aligned}(\hat\psi-\psi)\, j_p^{1/2}(\hat\psi) &= r_e(\psi)\\ &\dot\sim\ N(0,1) \text{ under } f(y;\theta)\end{aligned}}$$

$\ell_p'(\psi)\, j_p^{-1/2}(\hat\psi) = r_u$

$\dot\sim\ N(0,1)$ under $f(y;\theta)$

↓ all 6

glm(r ~ ., data = urine, family = binomial)

Coefficients

	Est $\hat\psi$	Std E $j_p^{-1/2}(\hat\psi)$	z $\hat\psi / (j_p^{-1/2})$	p-val
constant				
gravity				
ph	-0.49	0.57	-0.87	0.38
osmo				
$\vdots$				

#6 Calc Under $f(y; \psi=0, \underline{\lambda})$ obs $\doteq .0012^{***}$

from a std. Normal

Null deviance	105.17	on	76
residual deviance	57.56	on	70

$$2\left(\ell(\hat{\underset{\sim}{\theta}}) - \ell(\underset{\sim}{\theta}_0)\right) \,\dot\sim\, \chi^2_6 \qquad \theta = (\theta_1, \dots, \theta_6)$$

under $f(y; \underline{\theta}_0)$ $\left(\theta_0 = \underset{\sim}{0} \right)$

If we fit just 5 covariates

new resid. deviance & then difference

is $W_p = 2\{\ell_p(\hat{\psi}) - \ell_p(\psi)\} \,\dot\sim\, \chi^2_1$

if $\psi = 0$

$\psi = (\theta_6)$

- Using either r_e or r_p to test if $\psi = 0$.

- Confidence interval for ψ

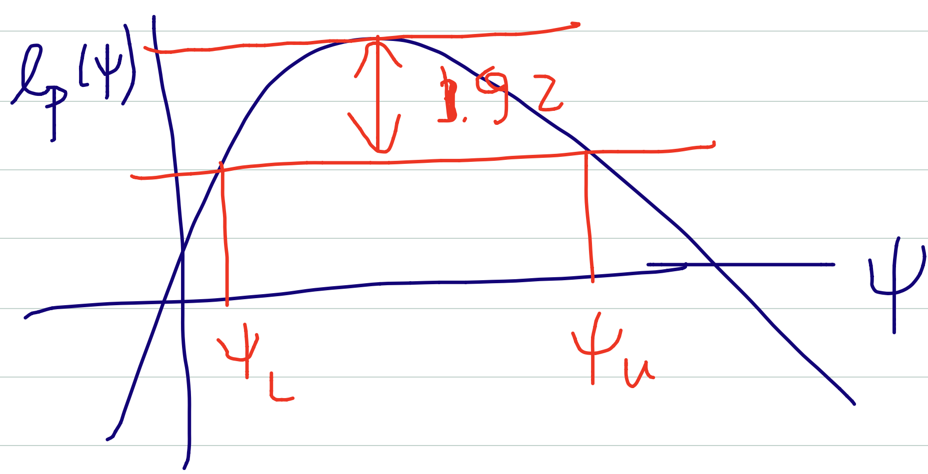
i) $\hat{\psi} \pm 1.96 \times \hat{s.e.}(\hat{\psi}) \approx 95\%$ CI

$\uparrow$ normal

ii) $w_p(\psi) = 2\{\ell_p(\hat{\psi}) - \ell_p(\psi)\} \mathrel{\dot\sim} \chi^2_1$

$\{\psi : w_p(\psi) \stackrel{\cdot}{\leq} 3.84\}$ 95% CI for ψ

$Pr(\chi^2_1 \leq 3.85) \doteq 0.95$



Effects of param. $\perp$

$$\hat\lambda_\psi - \lambda = \hat\lambda - \lambda + \overbrace{i_{\lambda\lambda}^{-1}\, \underbrace{i_{\lambda\psi}}_{0}\, (\hat\psi - \psi)}^{O_p\left(\frac{1}{\sqrt{n}}\right)} \boxed{+\, o_p\left(\frac{1}{\sqrt{n}}\right)}$$

$$\ell_\lambda(\psi, \hat\lambda_\psi) = 0$$

$$\doteq \cancelto{0}{\ell_\lambda(\hat\psi, \hat\lambda)} + (\psi - \hat\psi)\,\ell_{\lambda\psi}(\hat\psi, \hat\lambda)$$

$$+ (\hat\lambda_\psi - \hat\lambda)\,\ell_{\lambda\lambda}(\hat\psi, \hat\lambda)$$

$$(\hat\lambda_\psi - \hat\lambda) \doteq -\hat\ell_{\lambda\lambda}^{-1}(-\hat\ell_{\lambda\psi})(\hat\psi - \psi)$$

$$= i_{\lambda\lambda}^{-1}\, i_{\lambda\psi}\, (\hat\psi - \psi) + o_p(1)$$

If $\hat\lambda_\psi \equiv \hat\lambda$, then constant in ψ

$\ell_p(\psi, \hat\lambda_\psi) \qquad \ell(\psi, \hat\lambda)$

If $\hat{\lambda}_\psi$ varies slowly in ψ,
then inference based on profile
is closer to its limiting form.

To make precise, need to do
higher order expansions.

$$\begin{pmatrix} \hat{\psi}-\psi \\ \hat{\lambda}-\lambda \end{pmatrix} \dot{\sim} N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} i^{\psi\psi} & i^{\psi\lambda} \\ i^{\lambda\psi} & i^{\lambda\lambda} \end{pmatrix}\right)$$

$i_{\psi\lambda} = 0$ then $i^{\psi\lambda} = 0$ &
$\hat{\psi}$ $\hat{\lambda}$ a. independent

Example

BNC: $Y_i \sim N(\mu_i, \sigma^2)$

i) $\mu_i = \alpha + \beta x_i \qquad \mu_i = \gamma + \delta(x_i - \bar{x})$

$\uparrow$ $\qquad$ $\boxed{\gamma \perp \delta}$

$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x} \qquad \hat{\gamma} = \bar{y}$

$$\hat{\beta} = \frac{\Sigma(x_i - \bar{x})y_i}{\Sigma(x_i - \bar{x})^2} = \hat{\delta}$$

$y_i = x_i^T \beta + \sigma e_i \qquad e_i \sim N(0,1)$

$$i(\beta, \sigma^2) = \begin{pmatrix} \frac{X^T X_{p \times p}}{\sigma^2} & 0 \\ 0 & \frac{n}{2\sigma^4} \end{pmatrix} \leftarrow \text{ntbc}$$

$\beta \perp \sigma^2$ always for N

$X^T X$ is diag. then $\beta_j \perp \beta_k$

$$\hat{\sigma}_\beta^2 = \frac{1}{n}\sum_{i=1}^{n}(y_i - x_i^T\beta)^2$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum(y_i - x_i^T\hat{\beta})^2$$

$$= \hat{\sigma}_\beta^2 + \text{something}$$

$O_p(\frac{1}{n})$ check

$l_p(\psi) \leftarrow$ behaves 'like' a usual log-likelih. (to 1st order)

BUT $\hat{\psi} \not\xrightarrow{p} \psi$ $K \to \infty$

$\Rightarrow l_p(\psi)$ poor if $\dim(\lambda)$ "large" (too concentrated around ψ)

HW 2 $\quad f(s_1 \mid \underset{\sim}{s}_2 ; \beta_1)$

in logistic regr.

$$f(\underline{y};\underline{\beta}) \propto \underbrace{f(s_1 \mid \underset{\sim}{s}_2 ; \beta_1)}_{\text{only } \beta_1} f(\underset{\sim}{s}_2 ; \underline{\beta}_2)$$

cond

exp'l fam.

density

$$f(\underline{y};\underline{\theta}) \propto \underbrace{f(s_1 ; \theta_1)}_{\text{marg.} \leftarrow \text{transf.}} f(s_2 \mid s_1 ; \underset{\sim}{\theta})$$

marg. $\leftarrow$ transf.