Classical hyp. testing

How to choose 'best' test statistic?

Want T to have small p under H_0
large H_0^c

i.e. need to know dist^n under H_0^c & H_0

<u>Setup</u> Model $\{f(y;\theta),\ \theta \in \Theta\}$ response $Y \sim f(\,;\,)$

$Y \in \mathcal{Y}$

$H_0 : \theta \in \Theta_0 \qquad H_1 : \theta \notin \Theta_0$

Partition sample space into 2 regions
R, R^c (usually $R \cup R^c = \mathcal{Y}$)

obs $y \in R$ "reject H_0"
$y \in R^c$ "reject H_1"/"do not reject H_0"

1) $\overset{\alpha}{pr_{H_0}}\{\text{reject } H_0\} = \Pr(Y \in R^c \mid H_0)$ mistake

2) $\underset{\beta}{pr_{H_1}}\{\text{do not rej.}\} = \Pr(Y \in R \mid H_1)$ mistake

1) false positive   2) false negative

sensitivity of a test	1−2)	$\Pr(\overset{\text{reject}}{+\text{ve test}} \mid \overset{H_1}{\text{disease}})$
specificity	1−1)	$\Pr(\overset{\text{do not rej.}}{-\text{ve test}} \mid \overset{H_0}{\text{no disease}})$
	$\underline{\text{Intbc}}$	

Fix α i) $\Pr(Y \in R^c \mid H_0) = \alpha$?

maximize $\Pr(Y \in R^c \mid H_1) = 1-\beta$

'power of the test'

Opt$^{\underline{n}}$: choose R to max power for fixed α

(SM 7.3.2, CH Ch 4 but

Lehmann & R TSH)

Example $Y_1, \dots, Y_n$ iid $N(\mu, \sigma^2)$

$H_0: \mu = \mu_0$ $\qquad H_1:$ ~~$\mu > \mu_0$~~ $\mu = \mu_1 > \mu_0$

~~$\alpha \approx Pr_{H_0}$~~ $\quad T = \dfrac{\sqrt{n}(\bar{y} - \mu_0)}{s} \qquad \dfrac{\sqrt{n}(\bar{y} - \mu_0)}{s}$

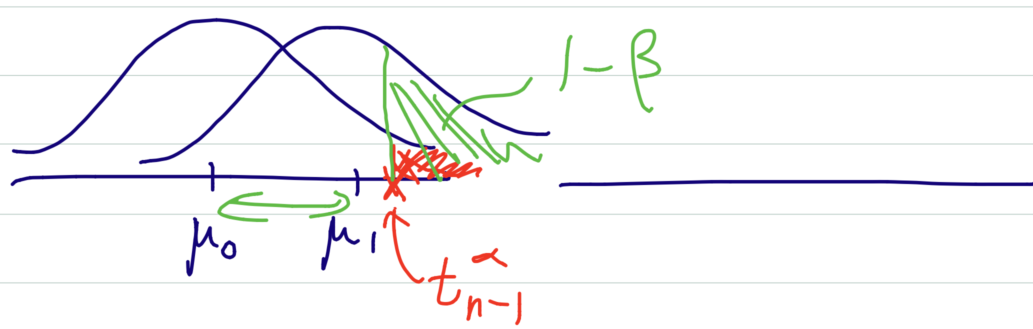
~~$\alpha \approx Pr \approx T \geq$~~ $R = \{ y;\ t(y) \geq t_{\alpha, n-1} \}$

where $t_{\alpha, n-1}$ is defined by t-distn
n-1 d.f.

$\alpha = 0.05 \quad n = 10 \quad \longleftarrow ?\ 2.32?$

$Pr_{\mu=\mu_0}(T \geq t^{\alpha}_{n-1}) = .05 = \alpha$ by constn

$1 - \beta = Pr_{\mu=\mu_1}(T \geq t^{\alpha}_{n-1})$ $\qquad Pr(\text{rej. } H_0 \mid H_1) = 1 - \beta$



If $H_0 : Y \sim f_0(y)$ $\quad H_1 : Y \sim f_1(y)$

MP test of level α of H_0 vs H_1 has

$$R = \left\{ y \,;\, \frac{f_1(y)}{f_0(y)} > c_\alpha \right\}$$

Choose R s.t.

$P_{H_0}\{Y \in R\} = \alpha$ $\quad Pr_{H_1}\{Y \in R\}$ is maximized

Define a test $f =$ $\phi(y) = \begin{cases} 1 & y \in R \\ 0 & y \notin R \end{cases}$

Concl.

Testing simple Null vs simple alt., best test statistic is likelihood ratio.

If we want to test more relevant hypotheses, we need more structure.

Example $\quad Y \sim \exp\{\theta s(y) - c(\theta) - d(y)\}$

$H_0: \theta = \theta_0 \qquad H_1: \cancel{\theta > \theta_0}\ \theta = \theta_1 > \theta_0$

NP Lemma:

$$R = \left\{ y : \frac{f_{\theta_1}(y)}{f_{\theta_0}(y)} > d_\alpha \right\}$$

$$\frac{e^{\theta_1 s(y) - c(\theta_1)}}{e^{\theta_0 s(y) - c(\theta_0)}} > d_\alpha$$

$$\left\{ y ; \quad \underbrace{(\theta_1 - \theta_0)}_{\text{+ve by ass}^{\underline{n}}} s(y) - \{c(\theta_1) - c(\theta_0)\} > D_\alpha \right\}$$

$$= \{ y ; \ s(y) > k_\alpha \}$$

1 example is $N(\mu, \sigma_0^2) \qquad s(y) = \bar{y}$

MP for all $\theta_1 > \theta_0$ UMP (uniformly)