STA 3000F Assignment 1.

1. Suppose that $Y_1, \ldots Y_n$ are i.i.d. from a Gamma distribution with shape parameter α and rate parameter λ:

$$f(y_i; \alpha, \lambda) = \lambda^\alpha y_i^{\alpha-1} e^{-\lambda y_i} / \Gamma(\alpha).$$

   Find the likelihood function for $\theta = (\alpha, \lambda)$ based on $(y_1, \ldots, y_n)$, and show that it depends on the data only through $\sum y_i$ and $\prod y_i$.

2. Suppose that Y_1 follows a $N\{\mu, \tau/(1-\rho^2)\}$ distribution, and that for $j = 2, \ldots, n$

$$Y_j = \mu + \rho(Y_{j-1} - \mu) + \epsilon_j$$

   where $(\epsilon_2, \ldots, \epsilon_n)$ are independently distributed as $N(0, \tau)$. Find the likelihood function for $\theta = (\mu, \tau, \rho)$ based on a sample $(y_1, \ldots, y_n)$. [See CH, Exercise 2.2; express the joint density as the product of conditional densities of successive observations given the past.]

3. Suppose that $Y_1, \ldots, Y_n$ are i.i.d from the density

$$f(y_i; \theta) = c(\theta) g(y_i), \quad 0 < y_i < \theta$$

   where $g(y)$ is a positive integrable function on $(0, \infty)$. Show that $Y_(n)$ is sufficient for θ. [TPE, Exercise 6.3]

4. Show that the Gamma(α, λ) distribution of Exercise 1. is a $(2, 2)$ exponential family. What is the canonical parameter?

5. **optional for MSc** Suppose we have observations from a one-way analysis of variance

$$Y_{jk} = \mu + \alpha_j + \epsilon_{jk}$$

   where $k = 1, \ldots r$, $j = 1, \ldots J$, $\alpha_j \sim N(0, \sigma_A^2)$, and $\epsilon_{jk} \sim N(0, \sigma^2)$, and the α_j's and ϵ_{jk}'s are mutually independent. Show that $\{\bar{Y}_{..}, \sum(\bar{Y}_{j.} - \bar{Y}_{..})^2, \sum(Y_{jk} - \bar{Y}_{j.})^2\}$ is sufficient for $\theta = (\mu, \sigma_A^2, \sigma^2)$. What is the minimal sufficient statistic if instead $k = 1, \ldots, r_j$, and all the r_j are different? [CH, Exercise 2.4]