STA 3000F Assignment 2.

1. Show that the Gamma(α, λ) distribution is a $(2,2)$ exponential family. What is the canonical parameter? [Question 4. from last week.]

2. Suppose that $Y_1, \ldots, Y_n$ are independent random variables, where Y_j takes the value 1 with probability p_j and 0 with probability $1 - p_j$. Assume that
$$\log \frac{p_j}{1-p_j} = \theta_0 + \theta_1 z_j$$
where $z_1, \ldots, z_n$ are known constants. Show that the joint distribution of $Y = (Y_1, \ldots, Y_n)$ is of exponential family form, with canonical parameters θ_0 and θ_1, and with minimal sufficient statistic $(\sum Y_j, \sum z_j Y_j)$. [Exercise 2.5 of CH; corrects Example 5.5 of TPE.]

3. Suppose Y has density (with respect to Lebesgue measure)
$$f(y; \eta) = \exp\{\sum_1^m \eta_i t_i(y) - c(\eta) - d(y)\}$$
and g is any differentiable function satisfying $E|g'(Y)| < \infty$, then
$$E\left[\left\{\frac{h'(Y)}{h(Y)} + \sum_1^m \eta_i t_i(Y)\right\} g(Y)\right] = -Eg'(Y),$$
where $h(y) = \exp\{d(y)\}$. [TPE, Lemma 5.15 and Exercise 5.18.]