STA 3000F Assignment 5.

1. Let $Y_1, \ldots, Y_n$ be independent, identically distributed from the Gamma distribution
$$f(y;\psi) = \frac{1}{\Gamma(\psi)} y^{\psi-1} e^{-y}, y > 0, \psi > 1.$$
Find and compare the forms of the standardized score statistic, the standardized Wald statistic, and the likelihood root.

2. In the two parameter Gamma model
$$f(y;\psi,\lambda) = \frac{1}{\Gamma(\psi)} \lambda^{\psi} y^{\psi-1} e^{-\lambda y},$$
find the profile likelihood for ψ and the form of the log-likelihood ratio statistic.

3. Let Y_{jk} be independent normal random variables with mean μ_j and variance σ^2, $k = 1, \ldots r; j = 1, \ldots n$. Find the maximum likelihood estimate of σ^2, and show that this estimate is not consistent as $n \longrightarrow \infty$, for fixed r.

4. Optional for MSc. In the components of variance model from Assignment 1:
$$Y_{jk} = \mu + \alpha_j + \epsilon_{jk},$$
$k = 1, \ldots r$, $j = 1, \ldots n$, $\alpha_j \sim N(0, \sigma_A^2)$, $\epsilon_{jk} \sim N(0, \sigma^2)$, suppose that the parameter of interest is the proportion of groups with positive mean $\mu + \alpha_j$. Show that the parameter of interest can be taken as $\psi = \mu/\sigma_A$. Obtain its profile loglikelihood.