STA 3000F Assignment 6.

1. (BNC 3.1): Let $Y_{ij}, i = 1, 2; j = 1, \ldots, r$ be independently normally distributed with known variance σ_0^2 and with $E(Y_{1j}) = \theta$, $E(Y_{2j}) = c\theta^2$, where c is a known constant. Obtain a reduction by sufficiency to means $(\bar{Y}_{1.}, \bar{Y}_{2.})$, showing membership of the $(2, 1)$ exponential family. Examine the form taken by the three test statistics

$$\begin{aligned} w(\theta_0) &= 2\{\ell(\hat{\theta}; y) - \ell(\theta_0; y)\} \\ w_u(\theta_0) &= \{\ell'(\theta_0)\}^2\{I(\theta_0)\}^{-1} \\ w_e(\theta_0) &= (\hat{\theta} - \theta_0)^2 I(\theta_0) \end{aligned}$$

where $I(\theta_0)$ is the Fisher information in the full sample from the model with θ equal to θ_0.

2. (Matched binary pairs): let $(Y_{1j}, Y_{2j}), j = 1, \ldots, m$ be independent pairs of independent binary random variables with $P(Y_{1j} = 1) = e^{\lambda_j}/(1 + e^{\lambda_j})$, $P(Y_{2j} = 1) = e^{\lambda_j + \psi}/(1 + e^{\lambda_j + \psi})$ where the parameter of interest is ψ. Show that the maximum likelihood estimate of λ_j for fixed ψ is

$$\hat{\lambda}_{j\psi} = \begin{cases} -\infty & (Y_{1j} + Y_{2j} = 0) \\ -\frac{1}{2}\psi & (Y_{1j} + Y_{2j} = 1), \\ +\infty & (Y_{1j} + Y_{2j} = 2). \end{cases}$$

Then show that the equation defining $\hat{\psi}$ is

$$1 + e^{\frac{1}{2}\hat{\psi}} = R_1/(\sum Y_{2j} - R_0),$$

where R_0, R_1 denote the numbers of pairs for which $Y_{1j} + Y_{2j} = 0$ or 1.

3. In the Gamma distribution

$$f(y; \psi, \lambda) = \frac{1}{\Gamma(\psi)} x^{\psi - 1} \lambda^{\psi} e^{\lambda x}$$

show that the parameter $\mu = EX$ is orthogonal to ψ with respect to (expected) Fisher information.