STA 3000F Assignment 7.

1. (CH, 4.2). The general non-randomized form of the Neyman-Pearson lemma is as follows. If $k_1(y), \ldots, k_{m+1}(y)$ are integrable functions over the sample space, not necessarily probability densities, then the region w which maximizes
$$\int_w k_{m+1}(y)dy,$$
subject to the constraints
$$\int_w k_j(y)dy = a_j \quad (j = 1, \ldots, m),$$
is defined by $\{y; k_{m+1}(y) \geq c_1 k_1(y) + \ldots + c_m k_m(y)\}$, provided that such constants $c_1, \ldots, c_m$ exist. Prove this result and use the result to find the critical region for the locally most powerful test of $H_0 : \theta = \theta_0$ against $H_A : \theta \neq \theta_0$ in the one-parameter model $f(y; \theta)$.

2. (CH, 4.7). Let $Y_1, \ldots, Y_n$ be i.i.d. with density $f(y; \theta) = \theta \exp(-\theta y)$. Obtain the uniformly most powerful test of $\theta = \theta_0$ against $\theta < \theta_0$ and derive the power function of the test.

3. (CH, 4.9). Let $Y_1, \ldots, Y_n$ be i.i.d. from a $N(0, \sigma^2)$ density and consider a test of $H_0 : \sigma^2 = 1$ vs. $H_A : \sigma^2 \neq 1$. Obtain the most powerful unbiased test.

4. Let $Y_1, \ldots, Y_n$ be i.i.d. with density
$$f(y; \theta) = \frac{1}{\theta}, 0 \leq y \leq \theta.$$
Obtain the most powerful test of $H_0 : \theta = \theta_0$ vs. $H_A : \theta = \theta_A$, where $\theta_A > \theta_0$, and show that the test is uniformly most powerful for $\theta > \theta_0$.