TA will return HW 2 on Thursday Mar 17

(no class otherwise after Tuesday)

Extra office Mar 21 Monday 3-5:00

Topics covered since last test

- generalized l. models - m.l. equations
  - iterative reweighted LS
  - estimate of dispersion ϕ
  - Gamma model (HW 2)
  - 2 eg's binomial, Poisson
- maximum likelihood inference (paper "Likelihood")

$$\hat{\underset{\sim}{\theta}} \mathrel{\dot\sim} N(\underset{\sim}{\theta}, j^{-1}(\hat{\underset{\sim}{\theta}}))$$

$$l'(\underset{\sim}{\theta}) \mathrel{\dot\sim} N(\underset{\sim}{0}, j(\hat{\theta}))$$

corr $\doteq$

where $j(\theta) = -\dfrac{\partial^2 l(\theta)}{\partial\theta\,\partial\theta^T}$

- def^n of likelihood, mle, observed & exp'd info
- asymptotic normality of mle
- likelihood ratio tests
- general purpose optimization / root-finding

1-dim: N-R, simple iteration, bracketing

p-dim: N-R, Nelder-Mead, BFGS

quasi-Newton

uniroot optim

nls nonlinear LS

Nonlinear Regression

$$y_i = \eta(\underset{\sim}{x}_i, \underset{\sim}{\beta}) + \varepsilon_i \qquad \eta(x_i, \underset{\sim}{\beta}) = E(y_i \mid x_i)$$

η known (hypothesized) nonlinear function

Ex. 1 : Olympic winning times

$$\eta(x_i, \underset{\sim}{\beta}) = \beta_0 + \beta_1 e^{\beta_2 x_i} \quad \beta_2 < 0 \quad \overset{H.O.}{(1)}$$

where x_i = year , y_i = time in that year

$\hookrightarrow \left(\frac{\text{year} - 1900}{4}\right)$

$$= \beta_0 + \beta_1 e^{-\beta_2 x_i} \qquad \beta_2 > 0$$

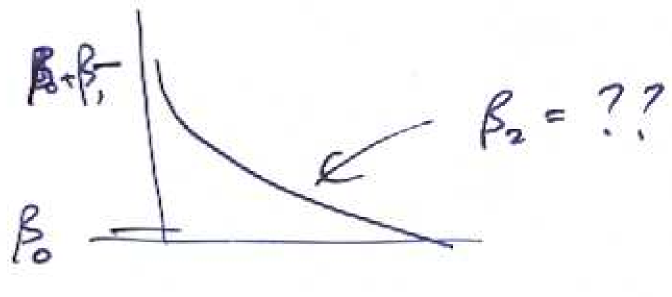
time in 1900 = $\beta_0 + \beta_1$

as $x_i \to \infty$ time $\to \beta_0$

β_0 theoretical best time

In R use nls(y ~ "formula", data = ... ,
start = c(b0 = _ , b1 = _ , ..)

?nls



$$y_i = \beta_0 + \beta_1 e^{\beta_2 x_i} + \varepsilon_i$$

$$\beta_0 \simeq 17 \qquad \beta_0 + \beta_1 \simeq 22$$

$$y_i - 17 = \beta_1 e^{\beta_2 x_i}$$

$$\log(\quad) = \log \beta_1 + \beta_2 x_i$$

<u>Ex. 2</u> x_i = amount of gamma radiation

y_i = growth (length) of new shoots (chick-peas)

$$\eta(x_i, \beta) = \beta_0 + \frac{\beta_1}{1 + e^{-\beta_2(x_i - \beta_3)}}$$

decreasing 'logistic growth model'

In general, let $\eta(x, \beta)$ be (expected) 'size' of a pop^n at 'time' x

$$\frac{\partial \eta(x)}{\partial x} = \underline{\gamma}\, \underline{\eta(x)} \left(\underline{\frac{\alpha - \eta(x)}{\alpha}} \right)$$

change in size = γ x current size x $\left(\frac{\alpha - \eta(x)}{\alpha}\right)$

α limiting value of $\eta(x)$ as $x \to \infty$

Sol^{n} $\eta(x) = \dfrac{\alpha}{1+\beta e^{-\gamma x}}$

$$\eta'(x) = \frac{-\alpha}{(1+e^{\beta e^{-\gamma x}})^2} \cdot \beta e^{-\gamma x} \cdot (-\gamma)$$

LHS $$= \frac{\gamma \alpha \beta e^{-\gamma x}}{(1+e^{\beta e^{-\gamma x}})^2}$$

$$RHS = \gamma \cdot \frac{\alpha}{(1+e^{\beta e^{-\gamma x}})} \cdot \left\{ 1 - \frac{1}{(1+e^{\beta e^{-\gamma x}})} \right\}$$

$$= \frac{\gamma \alpha e^{\beta e^{-\gamma x}}}{(1+e^{\beta e^{-\gamma x}})^2} \Leftarrow \text{what's } \beta?$$

at $x = 0$ $\eta(x) = \dfrac{\alpha}{1+\beta}$ ✓

$x \to \infty$ $\eta(x) \to \alpha$ ✓

Ex. 3 wt. loss over several days

y_i = weight

x_i = day

$$\eta(x_i, \beta) = \beta_0 + \beta_1 2^{-x_i/\beta_2}$$

$\beta_0 = \lim_{x_i \to \infty} \eta$ "ultimate cost"

β_1 total amount to be lost (eventually)

θ time to lose $\frac{1}{2}$ of remaining amount

$$y_i = \eta(x_i, \beta) + \varepsilon_i$$

often lots of theory to suggest a nonlinear model

?? $\varepsilon_i \sim \text{iid } N(0, \sigma^2)$??

OLS $\eta(x_i, \beta) = \underline{x}_i^T \beta$

optim : $\sum_{i=1}^{n} \{y_i - \eta(x_i, \beta)\}^2$