Exercises October 16 **STA 4508S (Fall, 2018)**

1. SM 4.9.2 Let $\eta(\theta)$ be a 1-1 transformation of θ, and consider a model with log-likelihoods $\ell(\theta)$ and $\ell^*(\eta) = \ell(\theta(\eta))$ in the two parametrizations respectively. Assume that $\ell(\cdot)$ has a unique maximum at which the score equation is satisfied.

   (a) Show that the Fisher information transforms as

   $$i^*(\eta) = \frac{\partial\theta^{\mathrm{T}}}{\partial\eta} i(\theta) \frac{\partial\theta}{\partial\eta},$$

   and that a similar equation holds for observed information $j(\hat{\theta})$, but not for the observed information function $j(\theta)$.

   (b) Show that the log-likelihood ratio statistic $w(\theta)$ and the standardized score statistic $w_u(\theta)$ are parametrization invariant, but that the standardized maximum likelihood statistic $w_e(\theta)$ is not, where

   $$\begin{aligned} w(\theta) &= 2\{\ell(\hat{\theta}) - \ell(\theta)\}, \\ w_u(\theta) &= U(\theta)^{\mathrm{T}} j^{-1}(\hat{\theta}) U(\theta), \\ w_e(\theta) &= (\hat{\theta} - \theta)^{\mathrm{T}} j(\hat{\theta})(\hat{\theta} - \theta). \end{aligned}$$

2. Suppose that $y_1, \ldots, y_n$ are independent, identically distributed random variables from an exponential family distribution

   $$f(y_i; \theta) = \exp\{\theta^{\mathrm{T}} s(y_i) - c(\theta) - d(y_i)\}.$$

   Show that the distribution of $S = \sum_{i=1}^n s(y_i)$ follows an exponential family distribution, with log-likelihood function

   $$\ell(\theta; s) = \theta^{\mathrm{T}} s - nc(\theta),$$

   and that the maximum likelihood estimate of θ is given by

   $$\mathrm{E}_{\hat{\theta}}(S) = s.$$

3. This is a journal reading exercise.

(a) Find a paper in one of the theoretical statistics journals (e.g. *Annals of Statistics, Biometrika, JASA Theory & Methods, JRSS B, Bernoulli*) with "likelihood" as one of the key words. Explain how likelihood is used in the paper.

(b) Find a paper in one of the applied statistics journals (e.g. *Annals of Applied Statistics, Biometrics, JASA applications, JRSS C*) with "likelihood" as one of the key words. Explain briefly how likelihood is used in the paper.