$$\ell(\theta;y) = \sum_{i=1}^{n} \ell(\theta;y_i) \qquad \boxed{\begin{aligned} U &= \ell' \\ &= \sum \ell_i' \end{aligned}}$$

$$\left[\ \ell(\theta;y_i) = \sum_{s} w_s \log f_s(y_s;\theta)\ \right]$$

$$(U_i \quad H \quad J)$$

$$\ell'(\hat{\theta}_{CL}) = 0 = U(\hat{\theta}_{CL}) \doteq U(\theta_0) + (\hat{\theta}_{CL} - \theta_0)\frac{\partial}{\partial\theta}U(\theta_0)$$

$$(X_n Y_n \xrightarrow{d} Xa)\ (\text{Slutsky})$$

$$\sqrt{n}(\hat{\theta}_{CL} - \theta_0) \doteq \frac{\frac{1}{\sqrt{n}} U(\theta_0)}{\underbrace{\left[-\frac{\partial}{\partial\theta} U(\theta_0)\frac{1}{n}\right]}_{\downarrow p\ H}} \doteq \underbrace{\frac{1}{\sqrt{n}} U(\theta_0)}_{\downarrow d\ N(0,\ J(\theta_0))} H^{-1}(\theta_0)$$

$$E_{\theta_0} U(\theta_0) = 0 \qquad E_{\theta_0} U U^{T} = J(\theta_0)$$

$$\hat{\theta}_{CL} - \theta_0 \doteq U_{CL}(\theta_0) H^{-1}(\theta_0)$$

$$\text{a.var}(\quad) \doteq H^{-1}(\theta_0) J(\theta_0) H^{-1}(\theta_0)$$

$$U(\theta_0) = \sum_{i=1}^{n} \underbrace{U(\theta_0; y_i)} \rightarrow \text{i.i.d.}$$

$$= \sum_{i=1}^{n} \left\{ \sum_{s \in S} w_s \frac{\partial}{\partial \theta} \log f(y_{is}; \theta) \Big|_{\theta=\theta_0} \right\}$$

$$\frac{1}{\sqrt{n}} U(\theta_0) \xrightarrow{d} N(0, \_\_)$$

$$\boxed{U(\hat{\theta}_{CL}) = 0}$$

true model $\overset{y_i \sim iid}{\downarrow}$ $f(\underline{y}; \theta_0)$ $\underline{y} \in \mathbb{R}^m$

$$\sqrt{n}(\hat{\theta}_{CL} - \theta_0) \xrightarrow{d} N(0, G^{-1}(\theta_0))$$

$$G(\theta_0) = H(\theta_0) J(\theta_0)^{-1} H(\theta_0)$$

$$\begin{array}{lll} \uparrow & \uparrow & \uparrow \\ -EU' & \text{var}\, U & -EU' \end{array}$$

$$E_{\theta_0} U(\theta_0) = 0$$
$$V(\quad) < \infty$$

$$l(\bar{\theta}_{CL}) = l(\hat{\theta}_{CL}) + (\bar{\theta} - \hat{\theta}_{CL}) \cancel{l'(\hat{\theta}_{CL})}^{0}$$

$$+ \frac{1}{2}(\theta - \hat{\theta}_{CL})^2 \, l''(\hat{\theta}_{CL})$$