



ELSEVIER

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

SCIENCE @ DIRECT®

Insurance: Mathematics and Economics xxx (2005) xxx–xxx

[www.elsevier.com/locate/ime](http://www.elsevier.com/locate/ime)

# Catastrophe options with stochastic interest rates and compound Poisson losses<sup>☆</sup>

Sebastian Jaimungal\*, Tao Wang

*Department of Statistics, University of Toronto, 100 St. George Street, Toronto, Canada M5S 3G3*

Received April 2005; received in revised form November 2005; accepted 24 November 2005

## Abstract

We analyze the pricing and hedging of catastrophe put options under stochastic interest rates with losses generated by a compound Poisson process. Asset prices are modeled through a jump-diffusion process which is correlated to the loss process. We obtain explicit closed form formulae for the price of the option, and the hedging parameters Delta, Gamma and Rho. The effects of stochastic interest rates and variance of the loss process on the option's price are illustrated through numerical experiments. Furthermore, we carry out a simulation analysis to hedge a short position in the catastrophe put option by using a Delta–Gamma–Rho neutral self-financing portfolio. We find that accounting for stochastic interest rates, through Rho hedging, can significantly reduce the expected conditional loss of the hedged portfolio.

© 2005 Elsevier B.V. All rights reserved.

*Keywords:* Catastrophe derivatives; Reinsurance; Compound Poisson process; Stochastic interest rates

## 1. Introduction

Although catastrophe derivatives have come into the limelight in recent years, little research has been published on the pricing and hedging issues associated with these complex instruments. Cox and Pedersen (2000) examine the pricing of catastrophe risk bonds, and briefly discuss the theory of equilibrium pricing and its relationship to the standard arbitrage-free valuation framework. Dassios and Jang (2003) use the Cox process, also known as the doubly stochastic Poisson process, to model the claim arrival process for catastrophic events; then they apply the model to the pricing of stop-loss catastrophe reinsurance contracts and catastrophe insurance derivatives. Gründl and Schmeiser (2002) analyze double trigger reinsurance contracts – a new class of contracts that has emerged in the area of “alternative risk transfer” – using several simplified modeling assumptions. In this paper, we focus on developing a coherent model for pricing and hedging catastrophe equity put options that are linked to both losses and share value of the issuing company.

In 1996 the first catastrophe equity put option or CatEPut<sup>1</sup> was issued on behalf of RLI Corp., giving RLI the right to issue up to US\$ 50 million of cumulative convertible preferred shares (Punter, 2001). In general, the CatEPut option gives the owner the right to issue convertible (preferred) shares at a fixed price, much like a regular put option; however,

<sup>☆</sup> This work was supported in part by the Natural Sciences and Engineering Research Council of Canada.

\* Corresponding author. Tel.: +1 416 946 8590; fax: +1 416 978 5133.

*E-mail address:* [sjaimung@utstat.utoronto.ca](mailto:sjaimung@utstat.utoronto.ca) (S. Jaimungal).

<sup>1</sup> CatEPut is a registered trademark of Aon Limited.

that right is only exercisable if the accumulated losses, of the purchaser of protection, exceed a critical coverage limit during the life time of the option. Such a contract, signed at time  $t$ , is a special form of a double trigger option and has a payoff at maturity of

$$\text{payoff} = \mathbb{I}_{\{L(T)-L(t) > \mathcal{L}\}}(K - S(T))_+ = \begin{cases} K - S(T), & S(T) < K \quad \text{and} \quad L(T) - L(t) > \mathcal{L}, \\ 0, & S(T) \geq K \quad \text{or} \quad L(T) - L(t) \leq \mathcal{L}, \end{cases} \quad (1)$$

where  $S(T)$  denotes the share value and  $L(T) - L(t)$  denotes the total losses of the insured over the time period  $[t, T]$ . The parameter  $\mathcal{L}$  is the trigger level of losses above which the CatEPut becomes in-the-money, while  $K$  represents the strike price at which the issuer is obligated to purchase unit shares if losses exceed  $\mathcal{L}$ .

In the event of a catastrophe, the share value of any insurance company that experiences a loss will also experience a downward jump. Consequently, it is likely that the embedded put option will end in-the-money and that the reinsurance company will be required to purchase shares at an unfavorable price. As such, it is prudent to develop a model which jointly describes the dynamics of the share value process and losses. Cox et al. (2004) were the first to introduce such a model for pricing catastrophe linked financial options. They assumed that the asset price process is driven by a geometric Brownian motion with additional downward jumps of a prespecified size in the event of a catastrophe. Since the life time of such an option can be 5 years or more, we generalize the results of Cox et al. (2004) to a stochastic interest rate environment. Moreover, their model assumes that only the event of a catastrophe affects the share value price while the size of the catastrophe itself is irrelevant. Such an assumption is a good first step; however, we propose that the loss sizes themselves should play a role, and therefore assume that the losses follow a compound Poisson process. Furthermore, we assume that the drop in asset price depends on the total loss level, rather than on only the total number of losses. When the losses, contingent on a catastrophe occurring, are a predetermined size and when the interest rates are constant, then our results reduce to those of Cox et al. (2004). In Section 2, we describe our joint asset, loss and interest rate model, in the real world, or statistical, measure, and then introduce a specific measure change to a particular risk-neutral measure.

The pricing of options in the presence of stochastic interest rates can generally be difficult. However, in Section 3 we make use of the forward-neutral measure to simplify the joint dynamics of interest rates and asset prices. This allows us to derive closed form formulae for the price of the CatEPut similar to those found by Cox et al. (2004). To understand the role that stochastic interest rates play, we conduct numerical experiments on the pricing equation in Section 4. The results indicate that stochastic interest rates can significantly affect prices for longer termed options. Our model also allows us to investigate the role that the additional stochasticity in the loss sizes themselves play. We find that the additional variance can either increase or decrease the value of the option depending on the size of the trigger level. This effect is explained by observing that the probability of exercising the option behaves in an analogous manner.

Pricing is only one aspect of the problem. It is also necessary to develop hedging strategies: This analysis is carried out in Section 5. Since the loss process itself is a non-tradable risk factor, we do not attempt to hedge the CatEPut directly. Instead we utilize a Delta, Gamma, Rho hedging scheme, which protects against changes in asset prices and interest rates. Since the asset price is correlated to losses, through the downward jumps in the asset price, it is possible to partially mitigate the tail risks through this scheme. By using simulation methods, we not only obtain the profit and loss distribution based on the prescribed hedging scheme but also illustrate the effects of model misspecifications.

## 2. The modeling assumptions

Let  $\{S(t) : t \geq 0\}$  denote the share value price process; let  $\{L(t) : t \geq 0\}$  denote the loss process of the insured; also let  $\{r(t) : t \geq 0\}$  denote the risk-free short rate process (or the force of interest). In addition, let  $\mathcal{F} \equiv \{\mathcal{F}_t : t \geq 0\}$  denote the natural filtration generated by these three processes, and let  $(\Omega, \mathbb{P}, \mathcal{F})$  represent the probability space with statistical probability measure  $\mathbb{P}$ . It is natural to assume that the interest rate process is stochastically independent of the loss process<sup>2</sup>; however, short rates and share value prices are typically negatively correlated. Our modeling assumption is

<sup>2</sup> We assume interest rate paths are continuous and, due to a fundamental result of Levy's, cannot be correlated with the jump component of the asset price process. Even if interest rate paths had discontinuous components, it is still reasonable to assume that interest rates and catastrophic losses are uncorrelated.

supplied by the system of SDEs:

$$S(t) = S(0) \exp\{-\alpha(L(t) - kt) + X(t)\}, \quad (2)$$

$$L(t) = \sum_{i=1}^{N(t)} l_i, \quad (3)$$

$$dX(t) = \left( \mu - \frac{1}{2}\sigma_S^2 \right) dt + \sigma_S d\tilde{W}^S(t), \quad (4)$$

$$dr(t) = \tilde{\kappa}(\tilde{\theta} - r(t)) dt + \sigma_r d\tilde{W}^r(t), \quad (5)$$

$$d[\tilde{W}^S, \tilde{W}^r](t) = \rho dt, \quad (6)$$

where  $\tilde{W}^S(t)$  and  $\tilde{W}^r(t)$  are correlated  $\mathbb{P}$ -Wiener processes driving the returns of the asset and the short rate respectively,  $\{l_i : i = 1, 2, \dots\}$  are i.i.d. random variables representing the size of the  $i$ -th loss with p.d.f.  $f_L(y)$  and mean  $\ell$ , and  $\{N(t) : t \geq 0\}$  is a homogenous Poisson process with arrival rate  $\lambda$ . The term  $kt$  is included explicitly in (2) to compensate for the presence of the downward jumps in share value price due to losses and is chosen such that

$$\mathbb{E}^{\mathbb{P}}[e^{-\alpha(L(t)-kt)}] = 1 \Rightarrow k = \frac{\lambda}{\alpha} \int_0^{\infty} (1 - e^{-\alpha y}) f_L(y) dy. \quad (7)$$

We have implicitly assumed that  $f_L(y) = 0$  for all  $y < 0$ . The parameter  $\alpha$  represents the percentage drop in the share value price per unit of loss and will be calibrated such that

$$\alpha \mathbb{E}^{\mathbb{P}}[l_i] = \delta \Rightarrow \alpha = \frac{\delta}{\ell}. \quad (8)$$

As argued in Cox et al. (2004), if a liquid market for CatEPut options exists, then standard derivative pricing theory implies an equivalent probability measure  $\mathbb{Q}$  exists, not necessarily unique, under which the discounted relative price processes  $\{\exp\{-\int_0^t r(s) ds\} \mathcal{S}(t) : t \geq 0\}$ , for all tradable assets  $\mathcal{S}(t)$ , are martingales (Harrison and Pliska, 1981). The measure  $\mathbb{Q}$  is known as the risk-neutral measure because the expected return of all tradable assets is equal to the risk-free rate. This measure change is by no means unique; however, we follow Cox et al. (2004) and make use of Merton's (1976) assumption that the jumps are non-diversifiable, and therefore the jump activity rate and distribution are not altered by the measure change.

**Proposition 2.1.** Let  $\eta(t)$  denote the Radon–Nikodym process

$$\begin{aligned} \eta(t) \equiv \left( \frac{d\mathbb{Q}}{d\mathbb{P}} \right)_t &= \exp \left\{ -\frac{1}{2} \int_0^t (\lambda_1^2(u, r(u)) + \lambda_2^2(u, r(u))) du \right. \\ &\quad \left. + \int_0^t \left( \lambda_1(u, r(u)) - \frac{\rho}{\sqrt{1-\rho^2}} \lambda_2(u, r(u)) \right) d\tilde{W}^r(u) + \int_0^t \frac{\lambda_2(u, r(u))}{\sqrt{1-\rho^2}} d\tilde{W}^S(u) \right\}, \end{aligned} \quad (9)$$

where the market prices of risk are

$$\lambda_1(u, r(u)) = \frac{1}{\sigma_r} [\kappa\theta - \tilde{\kappa}\tilde{\theta} + (\tilde{\kappa} - \kappa)r(u)], \quad (10)$$

and

$$\lambda_2(u, r(u)) = -\frac{\mu - r(u)}{\sigma_S \sqrt{1-\rho^2}} - \frac{\rho}{\sqrt{1-\rho^2}} \lambda_1(u, r(u)). \quad (11)$$

Then, for any  $\mathcal{A} \in \mathcal{F}_T$  we have that

$$\mathbb{Q}(\mathcal{A}) = \mathbb{E}^{\mathbb{P}}[\mathbb{I}(\mathcal{A})\eta(T)], \quad (12)$$

and in particular  $W^S(t)$  and  $W^r(t)$  defined by

$$W^S(t) = \tilde{W}^S(t) + \int_0^t \frac{\mu - r(s)}{\sigma_S} ds \quad (13)$$

$$W^r(t) = \tilde{W}^r(t) - \int_0^t \frac{(\kappa\theta - \tilde{\kappa}\tilde{\theta}) - (\kappa - \tilde{\kappa})r(s)}{\sigma_r} ds, \quad (14)$$

are  $\mathbb{Q}$ -Wiener processes with instantaneous correlation  $\rho$ ; i.e.  $d[W^S, W^r](t) = \rho dt$ . Finally, the discounted price process  $\{D(0, t)S(t) : t \geq 0\}$  is a  $\mathbb{Q}$ -martingale, here  $D(t, T) \equiv \exp\{-\int_t^T r(s) ds\}$ .

**Proof.** Clearly,  $\eta(t) > 0$   $\mathbb{P}$ -a.s.; furthermore, it is easy to see that  $\mathbb{E}^{\mathbb{P}}[\eta(t)] = 1$  and that  $\eta(t)$  is a  $\mathbb{P}$ -martingale. Consequently,  $\eta(t)$  is a Radon–Nikodym derivative process which induces the measure change from  $\mathbb{P}$  to  $\mathbb{Q}$ . Girsanov's theorem then implies that  $W^S(t)$  and  $W^r(t)$  are  $\mathbb{Q}$ -Wiener processes with instantaneous correlation  $\rho$ . Through straightforward calculations, we find that  $\{e^{-\int_0^t r(s) ds} S(t) : t \geq 0\}$  is a  $\mathbb{Q}$ -martingale.  $\square$

Note that the jump measure is unaffected by the measure change and that under  $\mathbb{Q}$  the joint asset, loss and interest rate dynamics can be written as

$$S(t) = S(0) \exp\{-\alpha(L(t) - kt) + X(t)\}, \quad (15)$$

$$L(t) = \sum_{i=1}^{N(t)} l_i, \quad (16)$$

$$dX(t) = \left(r(t) - \frac{1}{2}\sigma_S^2\right) dt + \sigma_S dW^S(t), \quad (17)$$

$$dr(t) = \kappa(\theta - r(t)) dt + \sigma_r dW^r(t), \quad (18)$$

$$d[W^S, W^r](t) = \rho dt. \quad (19)$$

The precise form of the measure change from  $\mathbb{P}$  to  $\mathbb{Q}$  does not have a material effect on the pricing issues ahead. The only requirements are (i) the distribution of losses and activity rate are known under the risk-neutral measure and (ii) the relative price process of the asset is a  $\mathbb{Q}$ -martingale. Therefore this assumption will be made throughout the remainder of this paper.

### 3. Pricing the catastrophe put

Under the risk-neutral measure, the value of CatEPut contracts can be obtained via discounted expectations. Letting  $C(t; t_0)$  denote the value of the option at time  $t$ , which was signed at time  $t_0 < t$ , and matures at time  $T > t$ , we have

$$C(t; t_0) = \mathbb{E}^{\mathbb{Q}}[D(t, T)\mathbb{I}_{\{L(T) > \mathcal{L} + L(t_0)\}}(K - S(T))_+ | \mathcal{F}_t]. \quad (20)$$

If interest rates are deterministic,<sup>3</sup> then the discount factor can be extracted from the expectation resulting in

$$C(t; t_0) = P(t, T)\mathbb{E}^{\mathbb{Q}}[\mathbb{I}_{\{L(T) > \mathcal{L} + L(t_0)\}}(K - S(T))_+ | \mathcal{F}_t] \quad (21)$$

where  $P(t, T)$  denotes the time  $t$  value of a  $T$ -maturity zero-coupon bond. In the presence of stochastic interest rates, a similar factorization can be obtained by performing a measure change to the forward-neutral measure  $\mathbb{Q}^T$ . This measure is defined by choosing the  $T$ -maturity zero coupon bond as the numeraire asset (Jamshidian, 1996). To this end, we now determine the dynamics of the asset price process under the forward-neutral measure  $\mathbb{Q}^T$ . It is well known that the price  $P(t, T)$  of a  $T$ -maturity zero coupon bond in the Vasicek model is of the affine form

$$P(t, T) = \exp\{A(t, T) - B(t, T)r(t)\}, \quad (22)$$

<sup>3</sup> Even if interest rates are uncorrelated to the stock price dynamics under the statistical measure, they will be correlated in the risk-neutral measure due to the presence of the short-rate in the drift of the asset price.

where

$$A(t, T) = \left( \theta - \frac{\sigma_r^2}{2\kappa^2} \right) (B(t, T) - (T - t)) - \frac{\sigma_r^2}{4\kappa} B^2(t, T), \tag{23}$$

$$B(t, T) = \frac{1}{\kappa} (1 - e^{-\kappa(T-t)}); \tag{24}$$

and the bond price satisfies the SDE:

$$\frac{dP(t, T)}{P(t, T)} = r(t) dt - \sigma_r B(t, T) dW^r(t). \tag{25}$$

Notice that the volatility term is deterministic.

**Proposition 3.1.** Let  $\beta(t)$  denote the Radon–Nikodym derivative process

$$\beta(t) \equiv \left( \frac{d\mathbb{Q}^T}{d\mathbb{Q}} \right)_t \equiv \frac{P(t, T)}{P(0, T)} D(0, t) = \exp \left\{ -\frac{1}{2} \int_0^t \sigma_r^2 B^2(u, T) du - \int_0^t \sigma_r B(u, T) dW^r(u) \right\}. \tag{26}$$

Then, for any  $\mathcal{A} \in \mathcal{F}_T$ ,

$$\mathbb{Q}^T(\mathcal{A}) = \mathbb{E}^{\mathbb{Q}}[\mathbb{I}(\mathcal{A})\beta(T)]. \tag{27}$$

In particular,  $\bar{W}^S(t)$  and  $\bar{W}^r(t)$  defined by

$$\bar{W}^S(t) = W^S(t) + \rho \sigma_r \int_0^t B(s, T) ds, \tag{28}$$

$$\bar{W}^r(t) = W^r(t) + \sigma_r \int_0^t B(s, T) ds, \tag{29}$$

are  $\mathbb{Q}^T$ -Wiener processes with instantaneous correlation  $\rho$ ; i.e.  $d[\bar{W}^S, \bar{W}^r](t) = \rho dt$ . Furthermore, if  $\mathcal{S}(t)$  is the price process of any tradable asset then  $\{\mathcal{S}(u)/P(u, T) : u \in [0, T]\}$  is a  $\mathbb{Q}^T$ -martingale.

**Proof.** Clearly,  $\beta(t)$  is a Radon–Nikodym derivative process that induces a measure change from  $\mathbb{Q}$  to  $\mathbb{Q}^T$ . A direct application of Girsanov’s theorem implies that  $\bar{W}^S(t)$  and  $\bar{W}^r(t)$  are  $\mathbb{Q}^T$ -Wiener processes. To prove the last statement, let  $\mathcal{S}(t)$  denote the price process of a tradable asset, and  $0 \leq t < u < T$ , then,

$$\mathbb{E}^{\mathbb{Q}^T} \left[ \frac{\mathcal{S}(u)}{P(u, T)} \middle| \mathcal{F}_t \right] = \frac{\mathbb{E}^{\mathbb{Q}}[(\mathcal{S}(u)/P(u, T))(d\mathbb{Q}^T/d\mathbb{Q})_u | \mathcal{F}_t]}{\mathbb{E}^{\mathbb{Q}}[(d\mathbb{Q}^T/d\mathbb{Q})_u | \mathcal{F}_t]} = \frac{\mathbb{E}^{\mathbb{Q}}[\mathcal{S}(u) D(0, u) | \mathcal{F}_t]}{\mathbb{E}^{\mathbb{Q}}[P(u, T) D(0, u) | \mathcal{F}_t]} = \frac{\mathcal{S}(t)}{P(t, T)}.$$

The last equality follows since (i)  $D(0, u) = D(0, t)D(t, u)$  and  $D(0, t) \in \mathcal{F}_t$  and (ii) both  $\mathcal{S}(u)$  and  $P(u, T)$  are tradable; therefore, their discounted versions are  $\mathbb{Q}$ -martingales. Finally, for every  $t \in (0, T]$  we have

$$\mathbb{E}^{\mathbb{Q}^T} \left[ \left| \frac{\mathcal{S}(t)}{P(t, T)} \right| \right] = \frac{\mathcal{S}(0)}{P(0, T)} < +\infty, \tag{30}$$

and the final statement follows.  $\square$

The above result holds true even though the stock price contains jumps. Another noteworthy point is the risk-neutral measure  $\mathbb{Q}$  and forward-neutral measure  $\mathbb{Q}^T$  coincide only when interest rates are purely deterministic ( $\sigma_r = 0$ ). As long as interest rate volatility is non-zero the two measures remain distinct, even if the correlation between interest rates and stock returns is zero: This is because the drift of the stock in the  $\mathbb{Q}$ -measure is equal to the short rate  $r(t)$  so that it is impossible for the stock price to be uncorrelated to interest rates under the  $\mathbb{Q}$ -measure.

**Theorem 3.2.** The price of the catastrophe equity put, assuming the risk-neutral dynamics given in (15)–(19), is

$$C(t; t_0) = e^{-\lambda(T-t)} \sum_{n=1}^{\infty} \frac{(\lambda(T-t))^n}{n!} \int_{\hat{\mathcal{L}}} f_L^{(n)}(y) \{KP(t, T)\Phi(-d_-(y)) - S(t) e^{-\alpha(y-k(T-t))} \Phi(-d_+(y))\} dy, \tag{31}$$

where  $f_L^{(n)}(y)$  represents the  $n$ -fold convolution of the loss probability density function  $f_L(y)$ ,

$$d_{\pm}(y) = \frac{\ln(S(t)/KP(t, T)) - \alpha(y - k(T - t)) \pm (1/2)\bar{\sigma}^2(t, T)}{\bar{\sigma}(t, T)}, \tag{32}$$

$$\bar{\sigma}^2(t, T) = \sigma_S^2(T - t) + \frac{2\kappa\rho\sigma_S\sigma_r + \sigma_r^2}{\kappa^2}((T - t) - B(t, T)) - \frac{\sigma_r^2}{2\kappa}B^2(t, T), \tag{33}$$

$\Phi(x)$  denotes the cumulative distribution function for a standard normal r.v., and

$$\hat{\mathcal{L}} = \max(\mathcal{L} + L(t_0) - L(t); 0). \tag{34}$$

**Proof.** From Proposition 3.1,  $\tilde{S}(t) \equiv S(t)/P(t, T)$  is a  $\mathbb{Q}^T$ -martingale. Through direct calculation, and on substitution of (28) and (29), we find

$$\tilde{S}(t) = \tilde{S}(0) \exp \left\{ -\alpha(L(t) - kt) - \frac{1}{2} \int_0^t \sigma^2(u, T) du + \int_0^t d\bar{Y}(u) \right\}, \tag{35}$$

where

$$\sigma(u, T) \equiv \sqrt{\sigma_S^2 + 2\rho\sigma_r\sigma_S B(u, T) + \sigma_r^2 B^2(u, T)}, \tag{36}$$

$$d\bar{Y}(u) = \sigma_S d\bar{W}^S(u) + \sigma_r B(u, T) d\bar{W}^r(u). \tag{37}$$

The option price is then obtained by computing the expectation in the forward-neutral measure,

$$C(t; t_0) = P(t, T) \mathbb{E}^{\mathbb{Q}^T} [\mathbb{I}_{\{L(T) > \mathcal{L} + L(t_0)\}} (K - \tilde{S}(T))_+ | \mathcal{F}_t], \tag{38}$$

where  $\tilde{S}(T) = S(T)$  since  $P(T, T) = 1$ . Conditioned on the number of losses observed over  $(t, T]$ , the distribution of  $\tilde{S}(T)$  is log-normal and writing

$$\tilde{S}(T) = \tilde{S}(t) \exp\{-\alpha(L(T) - L(t) - k(T - t)) + Z\} \tag{39}$$

then  $Z$  is a normal r.v. with

$$\mathbb{E}^{\mathbb{Q}^T} [Z | \mathcal{F}_t] = -\frac{1}{2} \bar{\sigma}^2(t, T), \tag{40}$$

$$\text{Var}^{\mathbb{Q}^T} [Z | \mathcal{F}_t] = \bar{\sigma}^2(t, T). \tag{41}$$

Therefore,

$$\begin{aligned} C(t; t_0) &= P(t, T) \mathbb{E}^{\mathbb{Q}^T} [\mathbb{I}_{\{L(T) > \mathcal{L} + L(t_0)\}} \mathbb{E}^{\mathbb{Q}^T} [(K - \tilde{S}(T))_+ | L(T)] | \mathcal{F}_t] \\ &= \mathbb{E}^{\mathbb{Q}^T} [\mathbb{I}_{\{L(T) > \mathcal{L} + L(t_0)\}} \{KP(t, T)\Phi(-d_-(L(T) - L(t))) \\ &\quad - S(t) e^{-\alpha(L(T) - L(t) - k(T - t))} \Phi(-d_+(L(T) - L(t)))\} | \mathcal{F}_t]. \end{aligned} \tag{42}$$

It then remains to compute the expectation over the observed losses  $L(T) - L(t)$ , and this leads directly to the final pricing result (31).  $\square$

Eq. (31) allows one to value the option when the distribution of losses is arbitrary. Although it appears that we require the  $n$ -fold convolution  $f_L^{(n)}(y)$  to all orders, in practice it is not necessary. When the activity rate  $\lambda$  of losses is small, only the first few terms of the summation over  $n$  in Eq. (31) have significant contributions to the result. This can be made more concrete by letting  $C(t; t_0; N)$  denote the approximation to the price by keeping only terms to order  $N$  in the summation. In that case, a crude upper bound for the estimated error in the price is given by

$$|C(t; t_0) - C(t; t_0; N)| < \frac{(\lambda(T - t))^N}{N!} (1 - e^{-\lambda(T - t)}) KP(t, T). \tag{43}$$

Consequently, the relative error in the price can be made as small as desired by choosing  $N$  appropriately. For example, if  $\lambda = 0.5$  (two catastrophes per year) with 5 years remaining in the life of the option, and with  $N = 10$ , an accuracy

of 0.3% can be obtained. Furthermore, the  $n$ -fold convolutions can be obtained for quite general loss distributions via very efficient and well known FFT methods.

#### 4. Numerical experiments

##### 4.1. Predetermined loss

The effects of stochastic interest rates can be measured by comparing our results with those of Cox et al. (2004). In addition to assuming constant interest rates, they assume that only the event of a loss decreases the share value of the insurance company, implying that the sizes of the claims are irrelevant. This assumption can be mimicked in our model by assuming that the loss size conditional on a loss is fixed at  $\ell$  and that the trigger level is an integer multiple of the loss size, i.e.  $\mathcal{L} = N\ell$ . The parameter  $N$ , which we coin the trigger ratio level, represents the ratio of the trigger level to the expected loss amount. A trigger ratio level of 1 represents the situation in which the trigger is achieved if expected losses are observed. Meanwhile, a trigger ratio level of  $N = 2$  represents that the trigger is achieved if twice the expected losses are observed, and so on. In this predetermined loss case, the probability density function of loss sizes is a Dirac density  $f_L(y) = \delta(y - \ell)$ , and the price function (31) collapses to

$$C(t; t_0) = e^{-\lambda\tau} \sum_{n=\hat{N}}^{\infty} \frac{(\lambda\tau)^n}{n!} \{KP(t, T)\Phi(-d_-(n\ell)) - S(t) e^{-\alpha(n\ell - k\tau)} \Phi(-d_+(n\ell))\}, \quad (44)$$

where  $\tau = T - t$ ,  $\hat{N} = \max(N - (N(t) - N(t_0)); 0)$  with  $(N(t) - N(t_0))$  equal to the number of catastrophes observed from  $t_0$  up to  $t$ , and the jump compensated drift is given by

$$k = \frac{\lambda}{\alpha}(1 - e^{-\alpha\ell}). \quad (45)$$

Note that Eq. (44) is quite similar in form to the results of Cox et al. (2004); however, the volatility is adjusted by the presence of stochastic interest rates. In the left panel of Fig. 1, we illustrate how the value of a 5-year CatEPut struck at  $K = 80$  varies as a function of the spot price  $S$  and the Poisson activity rate  $\lambda$ . Here, the percentage drop per loss  $\delta = 0.02$ , and the trigger ratio level  $N = 1$ . These contract parameters and the chosen model parameters allow a direct comparison of our results with those of Cox et al. (2004). In the right panel of Fig. 1, we plot the difference between our results with stochastic interest rates and those of Cox et al. (2004). Clearly, stochastic interest rates significantly affect the prices and induce at least a 20% price increase across all parameters.

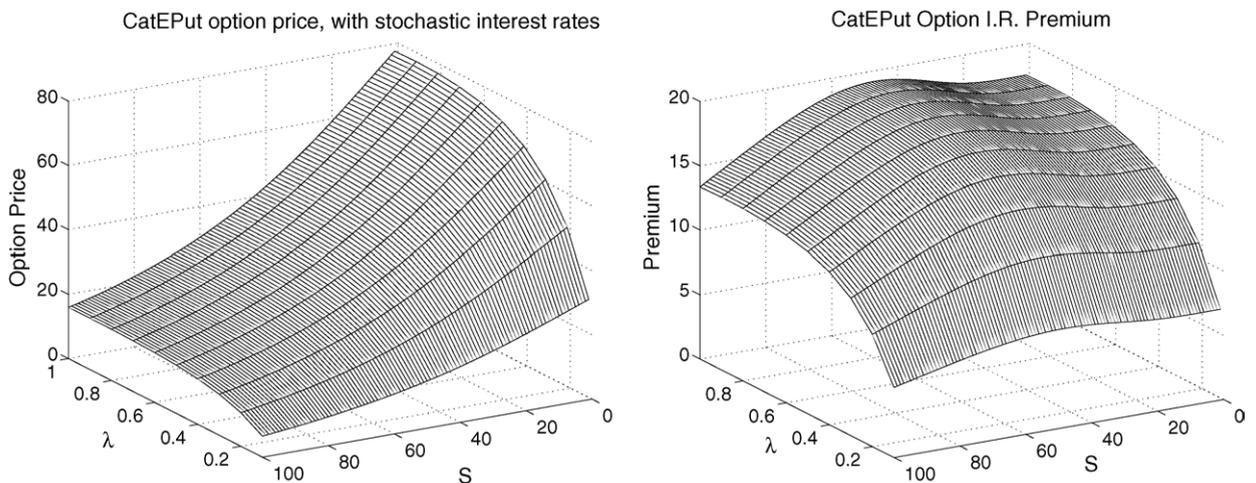


Fig. 1. This diagram depicts the value of the CatEPut contract as a function of spot price level and loss activity rate with a trigger ratio level of  $N = 1$  and a strike of  $K = 80$ . The right panel depicts the difference between prices when interest rates are stochastic and deterministic. The model parameters are  $r(0) = 2\%$ ,  $\kappa = 0.3$ ,  $\theta = 5\%$ ,  $\sigma_r = 15\%$ ,  $\sigma_S = 20\%$  and  $\rho = -0.1$ .

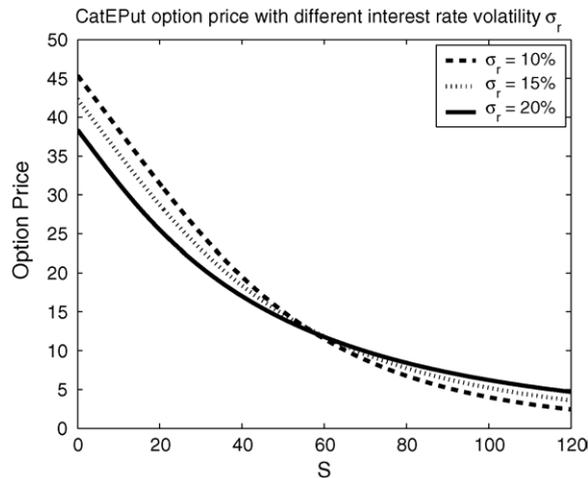


Fig. 2. This diagram depicts the value of the CatEPut contract for several interest rate volatility levels. The trigger ratio level  $N = 2$  and the strike level  $K = 80$ . The remaining model parameters are  $\sigma_S = 20\%$ ,  $\kappa = 0.3$ ,  $\rho = -0.1$  and  $\lambda = 0.5$ .

Next, in Fig. 2, we compare option prices with various levels of interest rate volatility using a trigger ratio level of  $N = 2$  and a strike price of  $K = 80$  on a 5-year term option. Note that we fix  $\kappa$  and adjust  $\theta$  so that the long-run spot interest rate

$$r_\infty \equiv \lim_{T \rightarrow \infty} \frac{\ln P(t, T)}{T - t} = \theta - \frac{\sigma_r^2}{2\kappa^2} \quad (46)$$

is a constant; in Fig. 2, we set  $r_\infty$  to 5%. There are two orthogonal factors which affect the option's price: (i) the discount effect and (ii) the uncertainty effect. The spot interest rates increase as volatility increases. Consequently, the cost of borrowing increases, pulling the option price downwards. In contrast, larger volatilities induce more uncertainty, and this pushes the option prices upwards. For small spot-levels, the discount effect dominates so that option prices decrease as volatility increases. For large spot-levels, the uncertainty is more significant, and option prices increase as volatility increases.

We also compare option prices with several mean-reversion rates in Fig. 3. In this case,  $\sigma_r$  is fixed and  $\theta$  is adjusted such that the long-run spot interest rate  $r_\infty$  remains at 5%. The two competing factors mentioned above also play a role here: spot interest rates decrease as the mean-reversion rate increases; as a result, the cost for borrowing money decreases, and option prices increase as mean-reversion rates increase. This factor dominates the competing factor of smaller fluctuations induced by higher mean-reversion rates. Nonetheless, there is a cross over effect at large spot-levels.

Finally, we compare the option price with different correlation coefficients in Fig. 4, which illustrates that prices decrease as the magnitude of the correlation coefficient increases. It appears that changes in  $\rho$  do not affect the prices as significantly as changes in the volatility or mean-reversion rates. The reason is if the asset price is strongly negatively correlated to interest rates, and asset prices are increasing, the option is moving deeper out-of-the-money and simultaneously the interest rates are decreasing, which increases the value of the option. On the other hand, as asset prices drop, and therefore the option is moving deeper in-the-money, interest rates increase (due to correlations being negative) and the value of the option decrease. These competing factors tend to smoothen out the prices as  $\rho$  changes.

All three results in Figs. 2–4 are consistent with the behavior of standard put option prices in a stochastic interest rate environment.

#### 4.2. Gamma losses

The deterministic loss case provides useful insight about the general behavior of the CatEPut; however, such a simplification cannot capture the role that uncertainty in the size of the losses plays. With this in mind, we now study the case in which loss sizes are drawn from a Gamma distribution. Specifically, let losses be distributed according to a Gamma distribution with mean  $\ell$  and stand deviation  $\sigma_L$ . The loss trigger level will be set equal to a multiple  $c$  of the

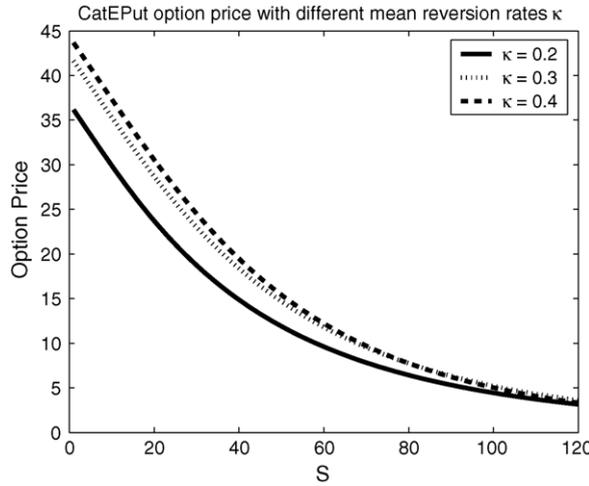


Fig. 3. This diagram depicts the value of the CatEPut contract for several mean-reversion rates. The trigger ratio level  $N = 2$  and the strike level  $K = 80$ . The remaining model parameters are  $\sigma_S = 20\%$ ,  $\sigma_r = 15\%$ ,  $\rho = -0.1$  and  $\lambda = 0.5$ .

expected losses over the term of the option

$$\mathcal{L} = c \mathbb{E}[L(T) - L(t)] = c \lambda (T - t) \ell, \tag{47}$$

where  $c$  is the real-valued trigger ratio level. The jump compensator drift in this case is

$$k = \frac{\lambda}{\alpha} \left( 1 - \left( \frac{\ell}{\ell + \alpha \sigma_L^2} \right)^{\ell^2 / \sigma_L^2} \right), \tag{48}$$

and the  $n$ -fold convolution of losses becomes

$$f_L^{(n)}(y) = \frac{y^{n\ell^2/\sigma_L^2 - 1} e^{-y/\sigma_L^2}}{(\sigma_L^2/\ell)^{n\ell^2/\sigma_L^2} \Gamma(n\ell^2/\sigma_L^2)}. \tag{49}$$

In Fig. 5, we illustrate how the option price is affected by the standard deviation of the claim size for two different trigger ratio levels. The option has a 5-year term, strike level of  $K = 80$ , and the mean loss size is set at  $\ell = 25$ . The diagram supplies the remaining model parameters. For a trigger ratio level of  $c = 1$  the option price decreases as the

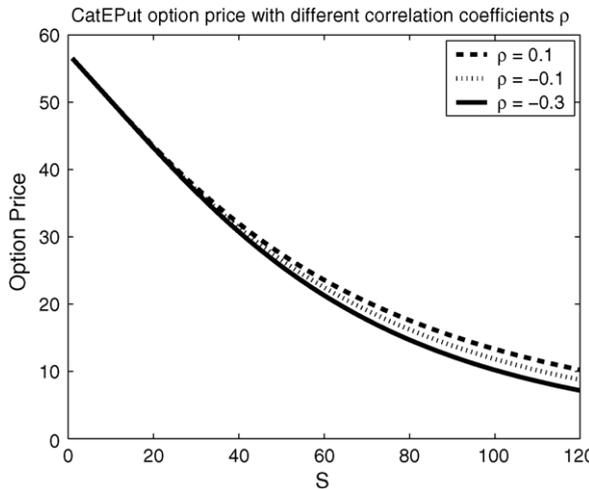


Fig. 4. This diagram depicts the value of the CatEPut contract with various correlation coefficients for  $N = 2$  and  $K = 80$ ,  $\sigma_S = 20\%$ ,  $\sigma_r = 15\%$ ,  $\kappa = 0.3$  and  $\lambda = 0.5$ .

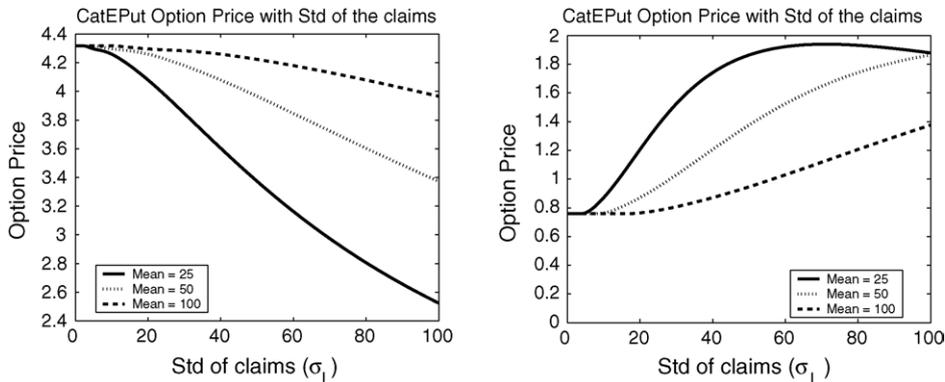


Fig. 5. This diagram depicts how the option price varies as a function of the standard deviation of claim sizes and of mean loss size for two trigger ratio levels  $c = 1$  (left panel) and  $c = 2$  (right panel). The strike of the option is  $K = 80$ . The remaining model parameters are  $\sigma_S = 20\%$ ,  $\sigma_r = 15\%$ ,  $\rho = -0.1$ ,  $\kappa = 0.3$  and  $\lambda = 0.5$ .

standard deviation of claim sizes increases; while, for a trigger ratio level of  $c = 2$ , the option price increases as the standard deviation of claim sizes increases. The second result is intuitively appealing; however, the first result seems somewhat paradoxical. This behavior is best explained by exploring the probability that the option is exercised. We omit the proof here as it follows along the same lines as [Theorem 3.2](#).

**Proposition 4.1.** *The real-world probability that the CatEPut option ends in the money is*

$$\mathbb{P}[S(T) < K, L(T) - L(t_0) > \mathcal{L} | \mathcal{F}_{t_0}] = e^{-\lambda(T-t_0)} \sum_{n=1}^{\infty} \frac{(\lambda(T-t_0))^n}{n!} \int_{\mathcal{L}} f_L^{(n)}(y) \Phi(h(y)) dy, \tag{50}$$

where

$$h(y) = \frac{\ln(K/S(t_0)) + \alpha(y - k(T - t_0)) - (\mu - (1/2)\sigma_S^2)(T - t_0)}{\sigma_S \sqrt{T - t_0}}, \tag{51}$$

and  $\mu$  is the real-world drift of the asset.

[Fig. 6](#) illustrates how the probability of exercise varies with the standard deviation of claims with  $\mu = 10\%$ . Notice that for smaller trigger ratio levels, the probability of exercising the option is generally a decreasing function of the standard deviation; while, for larger trigger ratio levels, it is an increasing function of the standard deviation. This observation reflects the behavior of the option prices observed in [Fig. 5](#).

**5. Dynamically hedging the catastrophe put**

Typically, when a reinsurer sells a CatEPut option to an insurance company, they will simultaneously hedge their position to avoid taking on very large losses. In this section, we will illustrate how to hedge against moderate movements in the asset price level and show how to hedge simultaneously against interest rate changes. In complete markets, asset price movements are hedged through Delta–Gamma hedging techniques that measure the sensitivity of the option’s price to asset price movements at the first and second order. Hedging against changes in interest rates is conducted through Rho hedging, which measures the sensitivity with respect to the short interest rate. Although this market is incomplete, such a strategy can be invoked to mitigate partially the risk. We can measure the strategy’s effectiveness by observing the profit and loss histogram generated by the strategy. Consider a CatEPut issued at time  $t_0$  with deterministic losses. The pricing equation for such a CatEPut is given in [\(44\)](#).

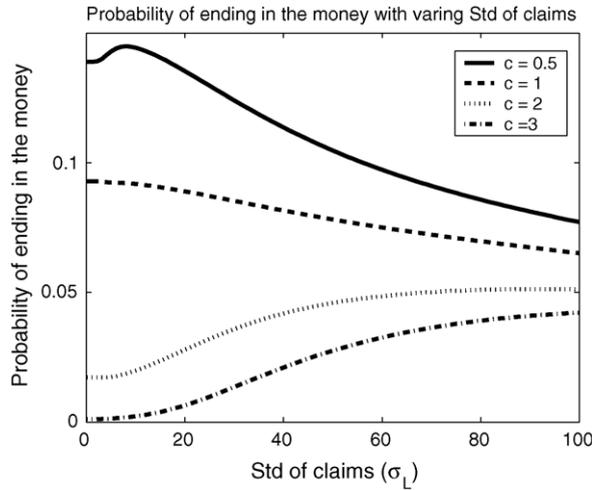


Fig. 6. This diagram depicts the probability of ending in-the-money with fixed mean  $\ell = 25$  and of increasing standard deviation of claims for several trigger ratio levels. The model parameters are  $\mu = 10\%$ ,  $\sigma_S = 20\%$  and  $\lambda = 0.5$ .

**Proposition 5.1.** *The Delta, Gamma and Rho of the CatEPut option with fixed loss sizes are given by the following expressions:*

$$\Delta(t; t_0) \equiv \frac{\partial}{\partial S(t)} c(t; t_0) = e^{-\lambda\tau} \sum_{n=\hat{N}}^{\infty} \frac{(\lambda\tau)^n}{n!} \{e^{-\alpha(n\ell - k\tau)} (\Phi(d_+(n\ell)) - 1)\}, \quad (52)$$

$$\Gamma(t; t_0) \equiv \frac{\partial^2}{\partial S^2(t)} c(t; t_0) = e^{-\lambda\tau} \sum_{n=\hat{N}}^{\infty} \frac{(\lambda\tau)^n}{n!} \left\{ e^{-\alpha(n\ell - k\tau)} \frac{\Phi'(d_+(n\ell))}{S(t)\bar{\sigma}(t, T)} \right\}, \quad (53)$$

$$\rho(t; t_0) \equiv \frac{\partial}{\partial r(t)} c(t; t_0) = e^{-\lambda\tau} \sum_{n=\hat{N}}^{\infty} \frac{(\lambda\tau)^n}{n!} \{B(t, T)P(t, T)(\Phi(d_-(n\ell)) - 1)\}, \quad (54)$$

where  $\tau = T - t$ ,  $\hat{N} = \max(N - (N(t) - N(t_0)); 0)$  with  $(N(t) - N(t_0))$  equal to the number of catastrophes observed from  $t_0$  up to  $t$ .

Although Delta–Gamma–Rho hedging is well known, we provide an overview of its application in the present context. We carry out the dynamic hedging strategy on a portfolio, consisting of a short CatEPut, and dynamically update units of the money-market account, the stock and two unique put options with strikes  $K_1$  and  $K_2$ . The portfolio will be rendered Delta, Gamma and Rho neutral at a finite number of rebalancing times  $t_i = i \Delta T$  for  $i = 1, \dots, m$  where  $T = m \Delta T$ . The CatEPut option is sold at time 0. Let

$$\pi \equiv \{a(t), b_1(t), b_2(t), h(t), -1\}_{t \in [0, T]} \quad (55)$$

denote the self-financing trading strategy, which the investor follows, where  $a(t)$  denotes the number of units of the asset,  $b_1(t)$  denotes the number of units of the put with strike  $K_1$ ,  $b_2(t)$  denotes the number of units of the put with strike  $K_2$  and  $h(t)$  denotes the number of units held in a risk-free money-market account, and one CatEPut is held short for the life time of the hedge. Note that the price, Delta, Gamma and Rho, of a standard put option can be obtained from the CatEPut price with the trigger level  $\mathcal{L}$  set to zero. Denote the price of a standard put of strike  $K_j$  by  $p_j(t)$ , the Deltas by  $\Delta_j(t)$ , the Gammas by  $\Gamma_j(t)$  and the Rhos by  $\rho_j(t)$ . The strategy at time  $t_i$  is obtained as follows: (i) ensure that the Gamma and the Rho of the two put options together match the Gamma and the Rho of the CatEPut (ii) introduce the stock to make the portfolio's Delta match the CatEPut's Delta and (iii) any funds required to purchase additional stock (if the change in delta is positive) are borrowed at the risk-free rate, or any excess funds obtained by selling stock (if the change in delta is negative) are invested at the risk-free rate. This results in the following self-financing strategy:

$$a(t_i) = \Delta(t_i; 0) - b_1(t_i)\Delta_1(t_i) - b_2(t_i)\Delta_2(t_i), \quad (56)$$

$$b_1(t_i) = \frac{\Gamma(t_i; 0)\rho_2(t_i) - \Gamma_2(t_i)\rho(t_i; 0)}{\Gamma_1(t_i)\rho_2(t_i) - \Gamma_2(t_i)\rho_1(t_i)}, \quad (57)$$

$$b_2(t_i) = -\frac{\Gamma(t_i; 0)\rho_1(t_i) - \Gamma_1(t_i)\rho(t_i; 0)}{\Gamma_1(t_i)\rho_2(t_i) - \Gamma_2(t_i)\rho_1(t_i)}, \quad (58)$$

$$h(t_i) = \frac{h(t_{i-1})}{P(t_{i-1}, t_i)} - (\Delta a(t_i)S(t_i) + \Delta b_1(t_i)p_1(t_i) + \Delta b_2(t_i)p_2(t_i)), \quad (59)$$

where  $\Delta a(t_i) \equiv a(t_i) - a(t_{i-1})$ , etc., and the initial amount invested in the Money-market account is

$$h(0) = c(0; 0) - (a(0)S(0) + b_1(0)p_1(0) + b_2(0)p_2(0)). \quad (60)$$

At the maturity date, if the asset price  $S(T) < K$  and the total losses is greater than the trigger level  $\mathcal{L}$ , then the CatEPut option will be exercised, and the holder of the long position will force the short holder to purchase shares at a price of  $K$ . After accounting for the possibility of exercise, the entire portfolio is liquidated at maturity, and the cash-flow is discounted back to the initial time.

To investigate the effectiveness of the hedging strategy, we perform simulations by generating 10,000 sample paths for the correlated diffusive components of the stock and interest rates, and the independent loss process. For each sample path, we also generate the corresponding antithetic path to reduce the variance of the simulation results. We can measure the effectiveness of the hedge by computing the conditional expected loss of the hedge (HCEL):

$$\text{HCEL} \equiv \mathbb{E}^{\mathbb{P}}[PnL - \mathbb{E}^{\mathbb{P}}[PnL]|PnL < \mathbb{E}^{\mathbb{P}}[PnL]], \quad (61)$$

where  $PnL$  represents the discounted value of the hedging strategy at maturity:

$$PnL \equiv \left( \prod_{i=1}^m e^{-r(t_i)\Delta t} \right) (a(t_m)S(t_m) + b_1(t_m)p_1(t_m) + b_2(t_m)p_2(t_m)) + h(t_m). \quad (62)$$

The measure (61) is similar to the conditional value-at-risk (CVAR) or short-fall measures used in portfolio risk management (Embrechts, 1999). CVAR is not only a coherent risk measure which allows us to compare the relative riskiness of two portfolios, but also is far superior to using standard deviation measures, especially in the context of asymmetric and kurtotic distributions. In the present context, we are comparing dynamic strategies that produce terminal PnL distributions which are typically asymmetric and kurtotic; therefore, it seems prudent to employ CVAR as a comparison tool.

We apply the following hedging scenarios to a 5-year CatEPut option with a strike of  $K = 80$ :

1. The real-world and hedging parameters are identical.
2. The real-world catastrophe frequency is higher than the hedging frequency ( $\lambda_{\text{real}} > \lambda_{\text{hedge}}$ ); all other parameters match.
3. The real-world percentage drop in share value is higher than the hedging percentage drop in share value ( $\alpha_{\text{real}} > \alpha_{\text{hedge}}$ ); all other parameters match.
4. The real-world interest rates are stochastic; while the hedger assumes constant interest rates, all other parameters match.

Using a monthly hedging frequency, the profit and loss histograms are displayed in Fig. 7 along with their descriptive statistics. The top right panel corresponds to case (1); the top left panel corresponds to case (2); the bottom left panel corresponds to case (3); the bottom right panel corresponds to case (4).

Case (1) is the optimal scenario as the hedger has full information about the real world dynamics and hedges accordingly. However, since the market is incomplete, and hedging is carried out in discrete time, the hedge is not exact. Nonetheless, there are no extremely large negative tails, as opposed to what is observed in the absence of hedging. Notice that the distribution is slightly positively skewed and has significant excess kurtosis while the mean is almost identically zero.

In cases (2)–(4), the hedger makes a model misspecification. In case (2), the hedger assumes that catastrophes occur only half as frequently as in the real world. However, it is not surprising that the distribution becomes more widely spread, but what is somewhat surprising is the bi-modal nature of the histogram. The mean is negative in this case

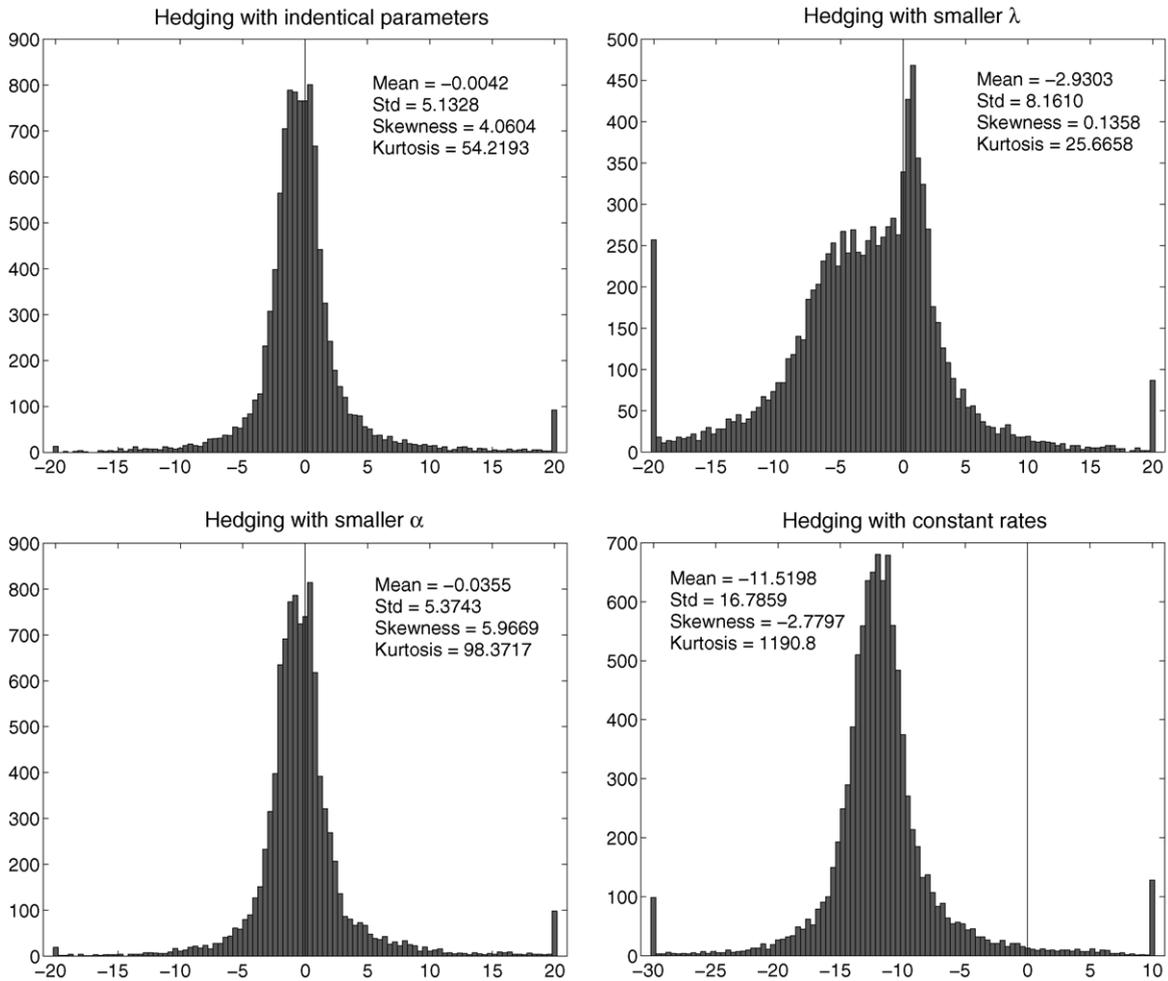


Fig. 7. This diagram depicts the discounted profit/loss distribution (62) under cases (1)–(4) (read from left to right, then top to bottom) as described in the body of the text. The contract parameters are  $K = 80$ ,  $K_1 = 88$ ,  $K_2 = 72$ ,  $S_0 = 80$ ,  $N = 1$  and  $T = 5$ . The model parameters are  $\alpha_{\text{real}} = 0.02$ ,  $\lambda_{\text{real}} = 0.5$ ,  $r(0) = 0.02$ ,  $\theta_{\text{real}} = 0.5$ ,  $\kappa_{\text{real}} = 0.3$ ,  $\sigma_{r(\text{real})} = 15\%$ ,  $\mu_{\text{real}} = 10\%$ ,  $\sigma_{S(\text{real})} = 20\%$ , and  $\rho_{\text{real}} = -0.1$ . In case (2),  $\lambda_{\text{hedge}} = 0.25$ ; in case (3),  $\alpha_{\text{hedge}} = 0.005$ ; in case (4),  $r_{\text{hedge}} = 0.05$ .

because the hedger charges too little for the option and pays the price of a heavier downside tail. In case (3), the hedger mistakenly assumes that shocks due to catastrophic events affect the stock price by half of what is actually realized. This particular misspecification seems to have the least adverse effect of all. In case (4), the hedger assumes that interest rates are constant and hopes that Rho hedging will be sufficient to cover interest rate changes. Firstly, such a hedger would significantly under price the option, which explains the negative mean of the distribution. Secondly, such a hedger would not correctly capture the volatility of interest rates and consequently, increase the spread of the profit and loss. Thirdly, the simulations show that skewness becomes negative.

In Fig. 8, the conditional expected losses of all four case are illustrated as the rebalancing frequency increases. In each case, increasing the rebalancing frequency positively affects the hedge and reduces the conditional expected loss. Interestingly, assuming that interest rates are constant, case (4) performs consistently worse than misspecifying the percentage drop due to losses in case (3) but performs consistently better than misspecifying the frequency of losses in case (2).

The simulation analysis carried out here is by no means exhaustive. However, it does illustrate the qualitative effects that our model predicts and further identifies the interest rate dynamics and frequency of losses as important factors in the hedging procedure.

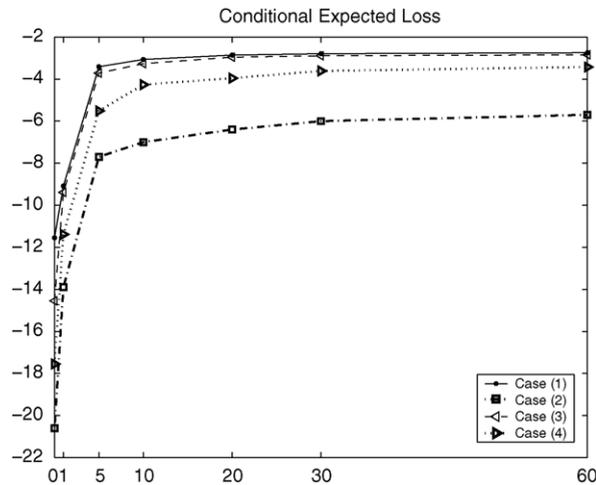


Fig. 8. This diagram depicts the conditional expected Loss in dollars today as a function of hedging frequency for cases (1)–(4) as described in the text and whose discounted profit/loss histograms are illustrated in Fig. 7.

## 6. Conclusions

In all, we have extended the analysis of Cox et al. (2004) to include further realism by introducing stochastic interest rates and stochastic claim sizes. Through the framework of a jump-diffusion model, we illustrate how catastrophic losses and an idiosyncratic diffusive component, together with correlated interest rate dynamics, affect option prices. Consequently, we successfully obtained closed form formulae for the price and various hedging parameters of the CatEPut option. Through numerical experiments and simulations, we demonstrated that both stochastic interest rates and the variance of the loss process play a significant role.

However, this work has several possible extensions and potential improvements. Firstly, the purchaser of the CatEPut is under significant counterparty risk from the seller of protection. Incorporating counterparty risk will certainly alter the pricing and hedging of the product; however, it is unclear what the magnitude of such a modification will be. Secondly, the CatEPut product may have an early exercise clause. Since these options are quite illiquid, the early exercise features pose non-trivial problems. One quick solution is to assume that the purchaser of protection will exercise their right as soon as the share-value drops below a given constant barrier level (once the losses have exceeded their trigger)—this is much like the method employed in employee stock option pricing. Thirdly, since catastrophes, such as hurricanes, can have a seasonality component, using an inhomogeneous driving Poisson process might lead to fruitful results. Since stochastic interest rates appear to play a significant role, the inclusion of stochastic volatility may also be an important factor to consider. Also, the model implementation can be improved by utilizing Fast Fourier Transform methods to evaluate the infinite sums appearing in Eq. (31). Finally, the model developed here can be applied to other structured risk management products, double-trigger stop-loss products for example. Gründl and Schmeiser (2002) study such products in a simplified context, using correlated log-normal index value and losses, and so we are currently applying an extension of our model to such products.

## Acknowledgements

The authors thank Sheldon X. Lin and an anonymous referee for useful comments and suggestions which ultimately enhanced the presentation of the paper.

## References

- Cox, S.H., Pedersen, H.W., 2000. Catastrophe risk bonds. *North American Actuarial Journal* 4, 56–82.
- Cox, H., Fairchild, J., Pedersen, H., 2004. Valuation of structured risk management products. *Insurance: Mathematics and Economics* 34, 259–272.
- Dassios, A., Jang, J.-W., 2003. Pricing of catastrophe reinsurance and derivatives using the Cox process with shot noise intensity. *Finance and Stochastics* 7 (1), 73–95.

- Embrechts, P., 1999. Extreme value theory as a risk management tool. *North American Actuarial Journal* 3 (2), 30–41.
- Gründl, H., Schmeiser, H., 2002. Pricing double-trigger reinsurance contracts: financial versus actuarial approach. *The Journal of Risk and Insurance* 69, 449–468.
- Harrison, J., Pliska, S., 1981. Martingales and stochastic integrals in the theory of continuous trading. *Stochastic Processes and Their Applications* 11, 215–260.
- Jamshidian, F., 1996. Bond and option evaluation in the Gaussian interest model. *Research in Finance* 9, 131–170.
- Merton, R., 1976. Option pricing when underlying stock returns are discontinuous. *Journal of Financial Economics* 3, 125–144.
- Punter, A., 2001. Securitisation of insurance risk. In: 2001 Hazard Conference, Aon Limited.