OPTION PRICING WITH REGIME SWITCHING LÉVY PROCESSES USING FOURIER SPACE TIME STEPPING

Kenneth R. Jackson
Department of Computer Science,
University of Toronto, Toronto, ON, Canada
krj@cs.toronto.edu

Sebastian Jaimungal
Department of Statistics and
Mathematical Finance Program,
University of Toronto, Toronto, ON, Canada
sebastian.jaimungal@utoronto.ca

Vladimir Surkov
Department of Computer Science,
University of Toronto, Toronto, ON, Canada
vsurkov@cs.toronto.edu

ABSTRACT

Although jump-diffusion and Lévy models have been widely used in industry, the resulting pricing partial-integro differential equations poses various difficulties for valuation. Diverse finite-difference schemes for solving the problem have been introduced in the literature. Invariably, the integral and diffusive terms are treated asymmetrically, large jumps are truncated and the methods are difficult to extend to higher dimensions. We present a new efficient transform approach for regime-switching Lévy models which is applicable to a wide class of path-dependent options (such as Bermudan, barrier, and shout options) and options on multiple assets.

KEY WORDS

Option Pricing, Lévy Processes, Regime Switching, Fourier Methods, American Options, Catastrophe Options.

1 Introduction

The option pricing problem under the classical model of Black and Scholes [3] and Merton [16] (BSM) reduces to solving a second-order parabolic PDE. This very successful PDE can be used to price a variety of options by imposing various terminal or boundary conditions, or employing early exercise constraints. It is well known that the BSM model is inconsistent with the behaviour of both price movements and the pricing of standard options such as calls and puts.

Jump-diffusion and Lévy models have been extensively applied in practice to account for the implied volatility smiles and term structures and partially correct the flaws in the BSM model. Under these models, the pricing PDE evolves into a PIDE with a non-local integral term. A plethora of finite difference methods for solving these PIDEs have been proposed in the recent literature, see e.g. [2], [4], [6], and [8]. Although the methods are quite diverse, they all have many points in common. The integral and diffusion terms of the PIDE are often treated separately. Invariably, the integral term is evaluated explicitly in order to avoid solving a dense system of linear equations. In addition, the Fast Fourier Transform (FFT) algorithm is employed to speed up the computation of the integral term (which can be regarded as a convolution) and/or its inverse.

Unfortunately, these methods require several approximations, such as: (i) in infinite activity processes, small jumps are approximated by a diffusion and incorporated into the diffusion term; (ii) the integral term must be localized to the bounded domain of the diffusion term, i.e. large jumps are truncated; (iii) the option price behaviour outside the solution domain must be assumed; and (iv) the separate treatment of diffusion and integral components requires that function values are interpolated and extrapolated between the diffusion and integral grids in order to compute the convolution term. These factors together make finite difference methods for option pricing under jump models quite complex, and potentially prone to accuracy and stability problems, especially for path dependent claims.

We present a new Fourier Space Time-stepping (FST) algorithm which avoids the problems associated with finite difference methods by transforming the PIDE into Fourier space. Working directly in Fourier space allows the characteristic exponent of an independent increment stochastic process to be factored out of the Fourier transform of the PIDE. The PIDE is then transformed into a linear system of easily solvable ordinary differential equations (ODE). Furthermore, the characteristic exponent is available, through the Lévy-Khintchine formula, in closed form for all independent increment processes. This makes the FST method quite flexible and generic – contingent claims on any exponential-Lévy stock price processes can then be priced with no additional modifications to the algorithm. The FST naturally leads to a symmetric treatment of the diffusion and jump terms and avoids any explicit assumptions on the option price outside of a truncated domain.

The remainder of this short article is organized as follows: Section 2 introduces the regime switching Lévy models of interest; Section 3 gives a very brief overview of our Fourier Space Time-Stepping methodology; Section 4 focuses on two interesting examples: American put options with regime switches and catastrophe equity put options; and Section 5 concludes with a discussion of ongoing and future work.

Details, more extensive numerical analysis and further examples can be found in our full paper [11].

2 The Model

Regime switching models can be traced back to the early work of Lindgren [14] and ever since the seminal work of Hamilton [9] [10] have become a very popular approach to incorporate non-stationary behaviour into an otherwise stationary model. The essential idea is to assume that the world switches between states representing, for example, moderate, low and high volatilities regimes. Such models have more explanatory power than conventional jump-diffusion or Lévy models and produce more realistic implied volatility smiles and term structures.

Although popular for describing time-series, little work has been carried out in terms of option valuation. Two-state European options in log-normal models were studied in Naik [18]; while European options in a two-state VG model were studied by Konikov and Madan [13]. Albanese, Jaimungal and Rubisov [1] derive closed form results for barrier and European options, and semi-closed form formulae for American options, in a special class of two-state VG models.

Let $\mathbb{K} := \{1, \ldots, K\}$ denote the possible hidden states of the world, and let $Z_t \in \mathbb{K}$ denote the prevailing state of the world at time t. We will assume that Z_t is driven by a continuous time Markov chain with generator A, i.e. the transition probability from state k at time t_1 to state l at time t_2 is $P_{kl}^{t_1 t_2} := \mathbf{Q}(Z_{t_2} = l | Z_{t_1} = k) = (\exp\{(t_2 - t_1)\mathbf{A}\})_{kl}$. The real matrix $\mathbf{A}$ satisfies the usual requirements: $A_{ll} = -\sum_{k \neq l} A_{lk}$ and $A_{kl} \geq 0 \ \forall k \neq l$. Given that $Z_t = k$, we assume that the joint stock price process $\mathbf{S}_t$ follows a d-dimensional exponential Lévy process with Lévy triple $(\boldsymbol{\gamma}^{(k)}, \mathbf{C}^{(k)}, \boldsymbol{\nu}^{(k)})$. This modeling assumption can succinctly be written $d\mathbf{X}_t = d\mathbf{X}_t^{(Z_t)}$, where $\mathbf{X}_t^{(k)}$ is the k-th d-dimensional Lévy process and the price processes are obtained by exponentiation component wise: $S_{j,t} = S_{j,0} \exp\{X_{j,t}\}$.

Recall that each Lévy process $\mathbf{X}^{(k)}$ admits a canonical Lévy-Itô decomposition

$$\begin{aligned}
\mathbf{X}_t^{(k)} &= \boldsymbol{\gamma}^{(k)}\, t + \mathbf{W}_t^{(k)} + \mathbf{J}_t^{l(k)} + \lim_{\epsilon \searrow 0} \mathbf{J}_t^{\epsilon(k)} \,, \\
\mathbf{J}_t^{l(k)} &= \int_0^t \int_{|y| \geq 1} \mathbf{y}\, \mu^{(k)}(d\mathbf{y} \times ds) \,, \\
\mathbf{J}_t^{\epsilon(k)} &= \int_0^t \!\! \int_{\epsilon \leq |y| < 1} \mathbf{y} \left[\mu^{(k)}(d\mathbf{y} \times ds) - \nu^{(k)}(d\mathbf{y} \times ds) \right] \,.
\end{aligned}$$

Here, $\mathbf{W}^{(k)}$ is a d-dimensional correlated Wiener process, $\mu^{(k)}(d\mathbf{y} \times ds)$ is a Poisson counting measure, which counts the number of jumps arriving in the interval $(s, s + ds]$ of size $(y_1, y_1 + dy_1] \times \cdots \times (y_d, y_d + dy_d]$. The vector of drifts $\boldsymbol{\gamma}^{(k)}$ in each state are assumed prefixed at their risk-neutral levels such that $\Psi^{(k)}(-i\mathbf{1}_j) = r$, for every j, where $\Psi^{(k)}(\boldsymbol{\omega})$ denotes the characteristic exponent of the k-th Lévy process

$$\begin{aligned}
\Psi^{(k)}(\boldsymbol{\omega}) \;=\; & \boldsymbol{\gamma}^{(k)} \cdot \boldsymbol{\omega} + \tfrac{1}{2}\, \boldsymbol{\omega} \cdot \mathbf{C}^{(k)} \cdot \boldsymbol{\omega} \\
& + \int_{\mathbb{R}^n} \left(e^{i\boldsymbol{\omega} \cdot \mathbf{y}} - 1 - i\mathbf{y} \cdot \boldsymbol{\omega}\; \mathbb{1}_{|\mathbf{y}|<1} \right) \nu^{(k)}(d\mathbf{y})
\end{aligned}$$

and $\mathbf{1}_j$ is the vector with zeroes everywhere except a single entry of 1 at dimension j.

Chourdakis [5] investigates the $d = 1$ version of this framework and derives the characteristic function of the terminal stock price. The author calculates European option prices via FFT methods; however, then resorts to numerical integration for the valuation of path-dependent options. We take a slightly different approach, and make use of transform methods which allows path-dependent options based on the regime switching models to be valued more efficiently.

Under the above assumptions, let $v(\mathbf{X}(t), Z(t), t)$ denote the discounted-adjusted and log-transformed price at time t conditional on the state $Z(t)$ and spot levels $\mathbf{X}(t)$. It is not difficult to show that European option prices satisfy the following system of PIDEs:

$$\left\{ \begin{array}{rl}
\partial_t + \left(A_{kk} + \mathcal{L}^{(k)} \right) v(\mathbf{x}, k, t) & \\
+ \sum_{j \neq k} A_{jk}\, v(\mathbf{x}, j, t) & = 0 \,, \\
v(\mathbf{x}, k, T) & = \varphi(\mathbf{S}(0) e^{\mathbf{x}}) \,,
\end{array} \right.$$

for every $k \in \mathbb{K}$. Here, $\mathcal{L}^{(k)}$ represents the infinitesimal generator of the k-th d-dimensional Lévy process:

$$\begin{aligned}
\mathcal{L}^{(k)} f(\mathbf{x}) \;=\; & \left(\boldsymbol{\gamma}^{(k)} \cdot \partial_{\mathbf{x}} + \tfrac{1}{2}\, \partial_{\mathbf{x}} \cdot \mathbf{C}^{(k)} \cdot \partial_{\mathbf{x}} \right) f(\mathbf{x}) \\
& + \int_{\mathbb{R}^n} \big(f(\mathbf{x} + \mathbf{y}) - f(\mathbf{x}) \\
& \qquad - \mathbf{y} \cdot \partial_{\mathbf{x}} f(\mathbf{x})\; \mathbb{1}_{|\mathbf{y}|<1} \big)\, \nu^{(k)}(d\mathbf{y}) \,.
\end{aligned}$$

It is possible in principle to apply any of the usual finite-difference schemes to the above system of PIDEs. However, as discussed earlier, this is quite difficult due to the non-local integral terms and especially so for multi-dimensional problems. Instead, we develop a transform algorithm in the next section.

3 The FST Algorithm

We begin by first discretizing the continuous time Markov chain by partitioning time into steps of size Δt and keep $Z(t)$ constant on time intervals $(t_{n-1}, t_n]$ (with $t_n = n\Delta t$), for $n \in \mathbb{N}$, with transition probabilities

$$P_{kl} := \left\{ \begin{array}{ll} 1 + A_{ll}\, \Delta t \,, & k = l \,, \\ A_{kl}\, \Delta t \,, & \mathit{otherwise} \,. \end{array} \right. \tag{1}$$

Then, by the martingale property of v and the law of iterated expectations we have

$$v(\mathbf{X}_{t_{n-1}}, Z_{t_{n-1}}, t_{n-1}) = \mathbb{E}^{\mathbb{Q}}_{t_{n-1}}\big[\mathbb{E}^{\mathbb{Q}}\left[v(\mathbf{X}_{t_n}, Z_{t_n}, t_n)|\mathcal{F}_{t_{n-1}} \vee Z_{t_n}\right]\big] \; . \quad (2)$$

Within the inner expectation, the process $\mathbf{X}(t)$ follows a given d-dimensional Lévy model, resulting in an expectation of the single regime form. In a given regime k, Jackson, Jaimungal and Surkov [11] show that

$$v^{n-1}(k) = \mathsf{FFT}^{-1}[\mathsf{FFT}[v^n(k)] \cdot e^{\Psi^{(k)}\,\Delta t}]$$

provides an efficient approximation to the price. Here, the dependencies on $\mathbf{X}$, Z and time have been suppressed for convenience and FFT represents the Discrete Fourier transform. Moreover, it is assumed that the log-stock space and frequency space have been discretized appropriately.

Due to the linearity of the (inverse) Fourier transform, the outer expectation in (2) can be computed to obtain the simple iterative scheme

$$v^{n-1}(j) \;=\; \sum_{k=1}^{K} P_{jk}\, \mathsf{FFT}^{-1}\left[\mathsf{FFT}[v^n(k)]\, e^{\Psi^{(k)}\,\Delta t}\right] \; .$$

At each time step, the algorithm therefore requires storing K prices. These K prices are then integrated backwards in time by the FST algorithm, then weighted according to the transition probabilities. If there are exercise decisions to be made, these must be made after averaging.

In this manner, the price of the option today in all K states will be known. Since the current state of the world is unknown to a trader, the trader must decide, exogenously, on a probability q_k that the world is in state k. Once these probabilities are determined, the trader's price for the option is $\sum_{k=1}^{K} q_k\, v(k)$.

4 Examples

Here we provide two prototypical applications of our pricing methodology.

The first example is an American put option pricing problem. Three world states are used. Within each state, the stock follows a Merton [17] jump-diffusion model in which jumps arrive at Poisson times, and are normally distributed.

The second example is an American catastrophe equity put option or CatEPut. Such options are triggered on when total losses to a company exceed a given threshold. Once the option is trigger on, its payoff is that of a regular put option. The stock and loss process are jointly modeled and only one world state is used to illustrate the effects of earlier exercise on these options.

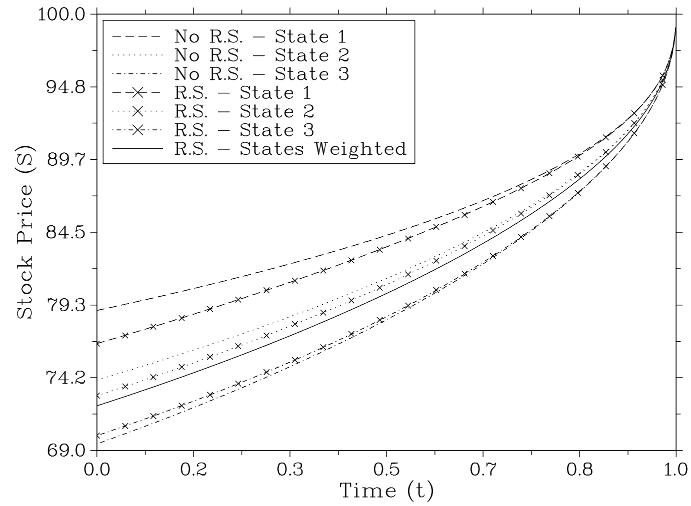

Figure 1. This diagram depicts the effect of regime switching on the exercise boundary of an American option. *Option:* American put $S = 100.0, K = 100.0, T = 1.0$; *Model:* Merton jump-diffusion $\sigma = 0.15, \tilde{\mu} = -0.5, \tilde{\sigma} = 0.45, r = 0.05$ *Regime Switching:* parameter states $\lambda \in [0.3, 0.5, 0.7]$, initial state probabilities $q = [0.2, 0.3, 0.5]$, Markov chain generator $A = [-0.8, 0.6, 0.2; 0.2, -1, 0.8; 0.1, 0.3, -0.4]$

N	M	Value	Change	$\log_2$ Ratio	Time (sec)
4096	256	14.250293			0.6
8192	512	14.250255	3.86×10^{-5}		4.7
16384	1024	14.250245	0.98×10^{-5}	1.98	14.4
32768	2048	14.250242	0.23×10^{-5}	2.11	63.4

Table 1. The convergence results for the American put option under Regime switches. The parameters are provided in Figure 1. The "$\log_2$-Ratio" column corresponds to the spatial convergence of the algorithm.

4.1 American Put Option with Three Regimes

As mentioned above, we first tackle the problem of pricing an American put option in which the price process follows a three state jump-diffusion model. In this case, the Lévy density in a given state k is

$$\nu(dy) = \tfrac{\lambda}{\sqrt{2\pi\tilde{\sigma}^2}} e^{-\frac{1}{2}((y-\tilde{\mu})/\tilde{\sigma})^2} \; ,$$

and the characteristic function is

$$\Psi(\omega) = i(\mu - \frac{\sigma^2}{2})\omega - \frac{\sigma^2\omega^2}{2} + \lambda(e^{i\tilde{\mu}\omega - \tilde{\sigma}^2\omega^2/2} - 1) \; .$$

For the numerical experiment, we assumed that only the arrival rate of jumps differs in the three regimes (low activity – state 1, moderate activity – state 2, high activity – state 3). The specific contract and model parameters are reported in Figure 1.

Figure 1 also shows the optimal exercise boundaries for the regime switching model conditional on being in a particular state. Furthermore, it contrasts the optimal boundaries with the boundaries obtained assuming the world remains in a given regime. Interestingly, in the regime with low activity, the optimal boundary shifts downwards when regime switching is turned off. Contrastingly, in the regime with high activity, the optimal boundary shifts upwards when regime switching is turned off. This reflects the intuition that the option behavior is pulled toward the mean level.

In Table 1 we present convergence results on the price of the option. The "log_2-Ratio" p reports the convergence power of the algorithm by measuring the price on successively finer grids. For this, we assume that $v_{appx} = v_{exact} + c(\Delta x)^p$ and estimate p via

$$p \approx \log_2 \frac{v_{appx}(\Delta x) - v_{appx}(\Delta x/2)}{v_{appx}(\Delta x/2) - v_{appx}(\Delta x/4)} .$$

From these results, we conclude the FST algorithm is order two in the spatial direction. We also found that by using Richardson extrapolation the method is order two in time.

Notice that the price with regime switching (14.2453) is lower than the price of the same American option when state 3 prevails (16.3071) for the entire option's lifetime. This is expected since the former model switches between periods of low, medium and high frequency of jumps, while the latter model always retains a high activity rate. Contrastingly, the price with regime switching is higher than the price when state 1 prevails (9.7144) for the entire option's lifetime. Moreover, the price is biased towards the high frequency price. This upward biasing occurs because the one-year transition probabilities for remaining in a given a state are approximately $[0.49, 0.46, 0.76]$ respectively. Consequently, in the regime switching model, the stock price processes remains in the high frequency state the longest.

4.2 CatEPut Options

Catastrophe Equity options have become more important in recent years, yet there is very little published on the subject. These options pay the holder a function of total losses and the company's equity value. A particularly popular option is the so called catastrophe equity put option (CatEPut). This option has payoff

$$\varphi(S(T), L_T) = \mathbb{1}_{L_T > U}(K - S(T))_+ .$$

In the event of large (catastrophic) losses above U, the insurer receives a put option on its own stock. These options are often exercisable any time prior to maturity.

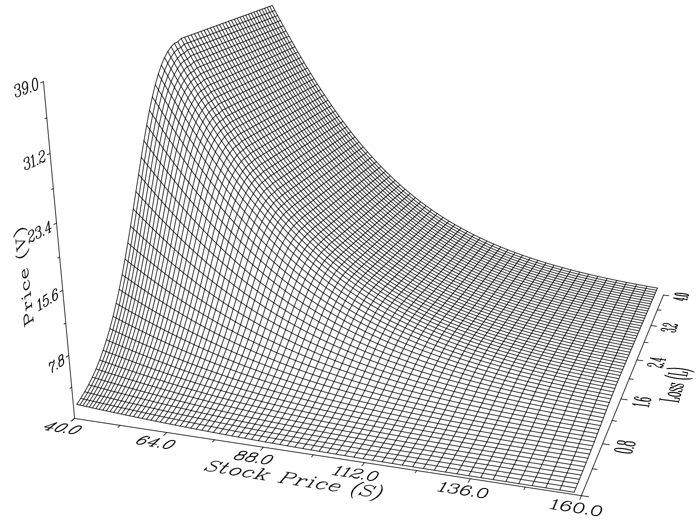

Figure 2. This diagram depicts the price of a European CatEPut option. *Parameters*: $T = 5$, $r = 0.05$, $\sigma = 0.2$, $U = 2$, $m = 0.2$, $v = 0.16$, $\lambda = 1$ and $\alpha = 0.05$.

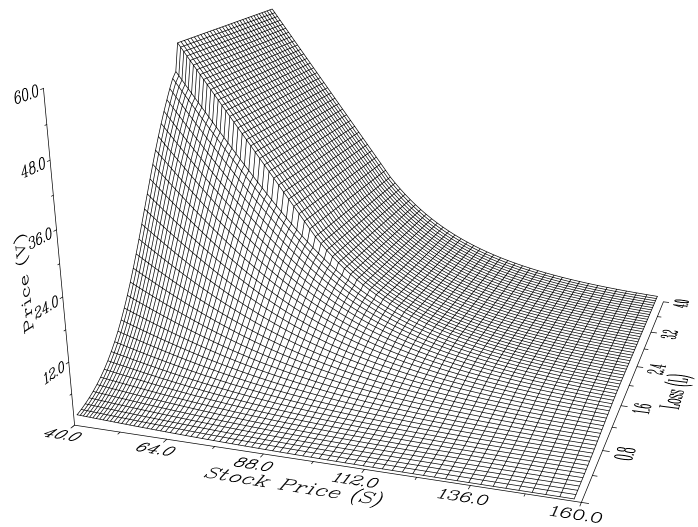

Figure 3. This diagram depicts the price of an American CatEPut option. *Parameters*: $T = 5$, $r = 0.05$, $\sigma = 0.2$, $U = 2$, $m = 0.2$, $v = 0.16$, $\lambda = 1$ and $\alpha = 0.05$.

Given the loss trigger in the payoff, it is important to jointly model losses and equity – large losses will cause significant drops in share value. Cox, Fairchild and Pedersen[7] introduce a simple model which counts the number of losses paying no attention to loss size. Jaimungal and Wang[12] extend the model to incorporate random losses as well as stochastic interest rates. However, both works assume that the option is European, while the existing contracts are in fact of American type.

Here, we illustrate how the FST algorithm can price the early exercise premium of the option efficiently. In the constant interest case, Jaimungal and Wang[12] make the following assumption on share value $S(t)$ and losses $L(t)$

$$\begin{aligned} S(t) &= S(0)\exp\left\{-\alpha\, L(t) + \gamma\, t + \sigma\, W_t\right\} \ , \\ L(t) &= \sum_{n=1}^{N(t)} l_i \ , \end{aligned}$$

where $N(t)$ is a compound Poisson process with activity rate λ, l_i are i.i.d. random variables with probability density $f_L(l)$ with support on $\mathbb{R}_+$, and W_t is a standard Brownian motion. Notice, it is the presence of losses which drives the jumps in the share value and not an independent jump process. This couples the share value with the loss process and highlights why such an option is popular: In the event of catastrophic losses, the company needs an influx of cash, such an event pushes the share value downward leading to a larger payoff of the embedded put option precisely when the holder needs it.

Loss sizes are often modeled as a Gamma random variable. Due to the coupling of share value and losses, the 2-dimensional Lévy density reduces to an effective one-dimensional Lévy density:

$$\nu(dy_1 \times dy_2) = f_L(y_2)\, \delta(y_1 + \alpha\, y_2)\, dy_1 dy_2 \ ,$$

resulting in a characteristic function for Gamma losses of

$$\begin{aligned} \Psi(\omega_1,\omega_2) \ &= \ i\,\gamma\,\omega_1 - \tfrac{1}{2}\sigma^2\,\omega_1^2 \\ &\quad + \lambda\left[\left(1 - i(-\alpha\,\omega_1 + \omega_2)\,\frac{v}{m}\right)^{-\frac{m^2}{v}} - 1\right] \ . \end{aligned}$$

Here, m denotes the mean loss size and v the variance of the loss size. As usual, the risk-neutral drift γ is set such that $\Psi(-i, 0) = r$.

For CatEPut options, the exercise policy must be chosen at each point in the $(S(t), L(t))$ plane independently. If $L(t)$ was not a separate observable, as it is in the usual jump-diffusion stock model, then the exercise policy would be independent of $L(t)$. When losses are below the trigger level U, the option is worthless, and therefore it is not optimal to exercise. However, since losses can only increase, when losses exceed U the option reverts to an American put option with downward jumps (due to the effect of losses on share value). The optimal policy is therefore analogous to an American put option when losses exceed U and depend only on the spot price.

In Figures 2 and 3, we provide the European and American CatEPut price as a function of spot price and loss level. These figures illustrate a few points of interests. Firstly, once losses are above the trigger level of U, the option reduces to a regular put option; however, the stock contains downward jumps, biasing prices higher than the put option on a GBM stock price process. For the European option, when losses are below the trigger level, there is a finite probability that the losses will rise, pushing the option into the money. Consequently, the price decreases smoothly as losses decrease. Contrastingly, there is a jump in the price of the American option at the trigger level U. This is due to the non-optimality of exercising when losses are below the trigger level.

It is important to note that these type of multi-dimensional options (with or without joint jumps in the underliers) can also be valued with regime switching incorporated. In the interest of brevity, this is left for future work.

5 Conclusions and Future Work

We introduced a new method for pricing European and path-dependent options when the underlying process(es) follows a regime switching Lévy process(es). The method treats the integral term and diffusion terms in the pricing PIDE symmetrically, is efficient, and accurate.

We demonstrated the appeal of the method by working through two very different prototypical examples: (i) an American put option in which the stock price undergoes regime changes, and (ii) an American CatEPut option in which the embedded put option is triggered on when losses exceed a critical level.

Pleasantly, Bermudan options can be priced using the FST method without any time stepping between exercise. This is a drastic computational advantage over finite-difference schemes and leading to more efficient and less biased pricing and hedging calculations.

Although we did not cover the application to barrier options in this article, we do explore their behavior in our longer article [11]. Barrier options do pose more challenges than American options due to the discontinuity in the option value created by the knock-out / knock-in provisions. In finite-difference methods, this problem is addressed by refining the mesh around the point of discontinuity. FFT algorithms, however, are designed for uniformly spaced points, making mesh refinement difficult to implement. Fortunately, FFTs using unequal spaced data (NFFT) have been developed recently (see e.g. Potts [19]) – the extension of the FST to use these methods is clear.

There are many open areas for exploration, such as: refinements of the American exercise boundary, efficient computation of the Greeks, parameter calibration, and stochastic optimal control problems.

As this work was completed, the very recent closely related work of Lord et.al. [15] came to our attention. We however, make the connection to the PIDE explicit, extend to multiple dimensions, and incorporate regime changes. Furthermore, we demonstrated how our method can value highly path dependent and non-standard options such as American catastrophe equity put options.

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