Chapter 12: From randomness to probability 350

Terminology

Sample space p351

The sample space of a random phenomenon is the set of all possible outcomes.

Example

Toss a coin.

Sample space: S = {H, T}

Example: Rolling a die.

S = {1, 2, 3, 4, 5,6}

Event p351

An **event** is an outcome or a set of outcomes of a random phenomenon.

That is, an event is a subset of the sample space.

Each occasion upon which we observe a random phenomenon is called a **trial**.

Example:

The sample space (S) for two tosses of a coin to is {HH, HT, TH, TT}.

Then exactly one head is an event, call it A, then A = {HT, TH}.

Independent trials

When thinking about what happens with combinations of outcomes, things are simplified if the individual trials are **independent**.

- Roughly speaking, this means that the outcome of one trial doesn't influence or change the outcome of another.

- For example, coin flips are independent.

The Law of Large Numbers p352

- The Law of Large Numbers (LLN) says that the relative frequency of some outcome reaches a limiting value as number of trials becomes large.

- We call the limiting value the **probability** of the event.

- That is, probability is a long-term relative frequency.

- Because this definition is based on repeatedly observing the event's outcome, this definition of probability is often called empirical probability.

Example: Tossing a coin: P(H) = ?

Theoretical probability

- Sometimes can argue (in a mathematical model) what probabilities should be:
  - toss a coin: two faces of a coin are just the same, so coin should be equally likely to land heads or tails, eg. P(H)=1/2.
  -
  - roll a die: in theory it is a perfect cube, so each of the 6 faces equally likely to be uppermost: eg. P(6)=1/6.
  - more generally, any time you have equally likely outcomes, prob. of event A is

$$P(A)=\frac{\text{Number of outcomes in } A}{\text{Number of outcomes in } S}$$

Example

Roll a red die and a green die. Find the probability of getting 10 spots in total.

- Sample space,(with the first number showing the number of spots on the red die first:
- S={(1,1),(1,2),...,(1,6),(2,1),...,(6,6)}
- all 36 possibilities equally likely.
- Which of those possibilities add up to 10?
- How many of them are there?

So what is the (theoretical) probability of total of 10?

Personal (subjective) probability

- What is probability that it will rain tomorrow? How does weather forecaster get “40%”?
- Forecaster uses experience to say that in “similar situations” in past, it's rained about 40% of the time.

- Personal probabilities not based on long-run behaviour or equally likely events. So treat with caution.

Probability rules p 355

1. The probability *P*(*A*) of any event *A* satisfies $0 \leq P(A) \leq 1$.

2. If *S* is the sample space in a probability model, then *P*(*S*) = 1.

3. The complement of any event *A* is the event that *A* does not occur, written as A^c. The complement rule states that

$$P(A^c) = 1 - P(A).$$

4. Two events A and B are **disjoint** if they have no outcomes in common and so can never occur simultaneously.

If A and B are disjoint,

$$P(A\, or\, B) = P(A) + P(B).$$

This is the **addition rule for disjoint events**.

- This can be extended for more than two events

General Addition rule for the unions of two events p358

For any two events A and B

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Ex: A retail establishment accepts either the American Express or the VISA credit card. A total of 24% of its customers carry an American Express card, 61% carry a VISA card, and 11% carry both.

What percentage of its customers, carry a card that the establishment will accept?

Ex: Among 33 students in a class 17 of them earned A’s on the midterm exam, 14 earned A’s on the final exam, and 11 did not earn A’s on either examination. What is the probability that a randomly selected student from this class earned A’s on both exams?

(Venn diagrams)

Chapter 13: Probability Rules p369

Conditional Probability 369

When $P(A) > 0$, the conditional probability of B given A is

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Ex: Here is a two-way table of all suicides committed in a recent year by sex of the victim and method used.

	Male	Female
Firearms	15,802	2,367
Poison	3,262	2,233
Hanging	3,822	856
Other	1,571	571
Total	24,457	6,027

(a) What is the probability that a randomly selected suicide victim is male?

There were 24,457 + 6,027 = 30,484 suicides altogether. $\frac{24{,}457}{30{,}484} = 0.8023$.

(b) What is the probability that the suicide victim used a firearm?

$$\frac{15{,}802+2{,}367}{30{,}484} = 0.5960$$

(c) What is the conditional probability that a suicide used a firearm, given that it was a man?

Among men: $\frac{15{,}802}{24{,}457} = 0.6461$

Given that it was a woman?

Among women: $\frac{2{,}367}{6{,}027} = 0.3927$

Independent events ***p371***

Two events A and B are both have positive probability are **independent** if

$$P(B|A)=P(B).$$

General multiplication rule p372

$$P(A \text{ and } B)=P(A)\times P(B|A)$$

$$P(A \text{ and } B)=P(B)\times P(A|B)$$

Another example

Law	acc.	rej.	total		Business	acc.	rej.	total
males	10	90	100		males	480	120	600
females	100	200	300		females	180	20	200
total	110	290	400		total	660	140	800

- Randomly select *two* male applicants to law school. What is probability that they are *both* rejected?
- *R1 event "1st one rejected"*
- *R2 event "2nd one rejected"*
- *P(R1 and R2)=P(R1) x P(R2|R1)=90/100 x 89/99*

Independence and disjointness

- If two events A, B are disjoint, they *can't both happen.*
- Suppose A happens, then P(B|A) **must be 0**, whatever P(B) is.

- Suppose now C and D are independent events.
- Then P(D|C) **equals** P(D): knowing about C *makes no difference.*

Turning conditional probabilities around

Suppose a restaurant has two (human) dishwashers. Alma washes 70% of the dishes, and breaks (on average) 1% of those. Kai washes 30% of the dishes, and breaks 3% of those. You are in the restaurant and hear a dish break at the sink. What is the probability that it was Kai?

The dishwashing example, another way

Suppose a restaurant has two (human) dishwashers. Alma washes 70% of the dishes, and breaks (on average) 1% of those. Kai washes 30% of the dishes, and breaks 3% of those. You are in the restaurant and hear a dish break at the sink. What is the probability that it was Kai?

Pretend there are 1000 dishes. Then Alma washes 700, and breaks 7 (700(0.01)) . Kai washes 300, and breaks 9 (300(0.03)). Make a table:

	Breaks	Does not break	Total
Alma	7	700-7=693	700
Kai	9	300-9=291	300
Total	16		1000

16 dishes were broken, 9 by Kai, so P(Kai|dish broke)=9/16.

“At least one”

Ex:

The gene for albinism in humans is recessive. That is, carriers of this gene have probability 1/2 of passing it to a child, and the child is albino only if both parents pass the albinism gene. Parents pass their genes independently of each other. If both parents carry the albinism gene, what is the probability that their first child is albino?

If they have two children (who inherit independently of each other), what is the probability that

a) both are albino?

b) neither is albino?

c)exactly one of the two children is albino?

d) If they have three children (who inherit independently of each other), what is the probability that at least one of them is albino?