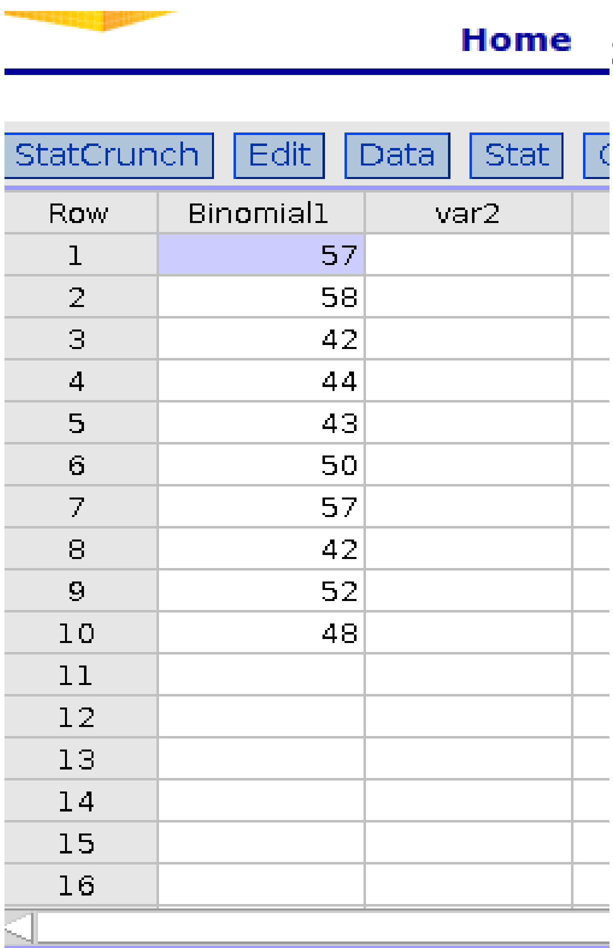
Chapter 15: Sampling Distribution Models

P 433

If you toss a fair coin 100 times, how many heads might you get?

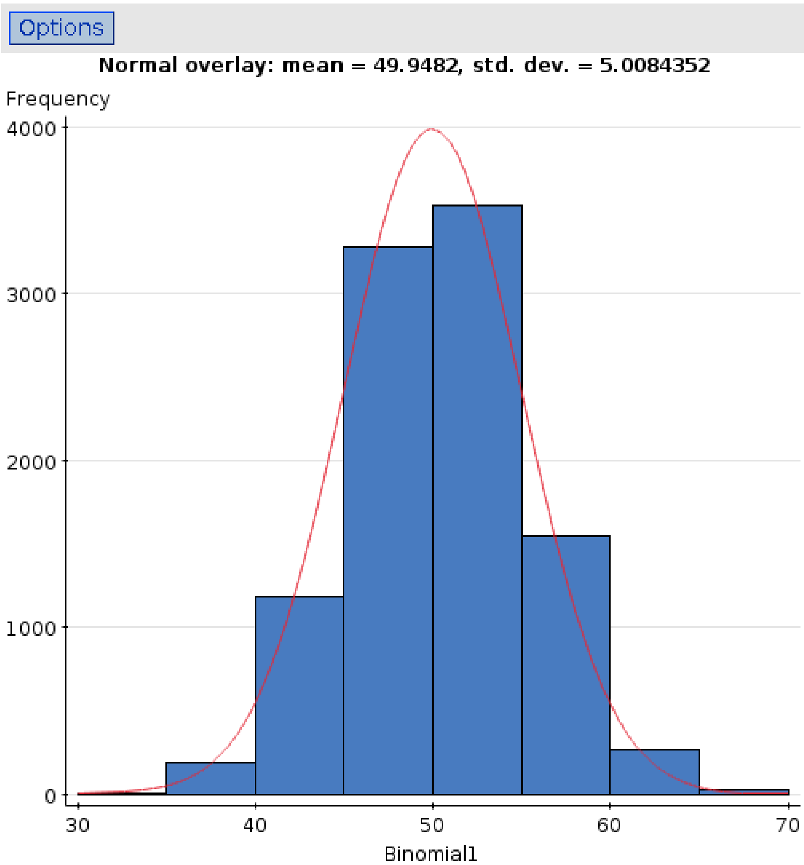


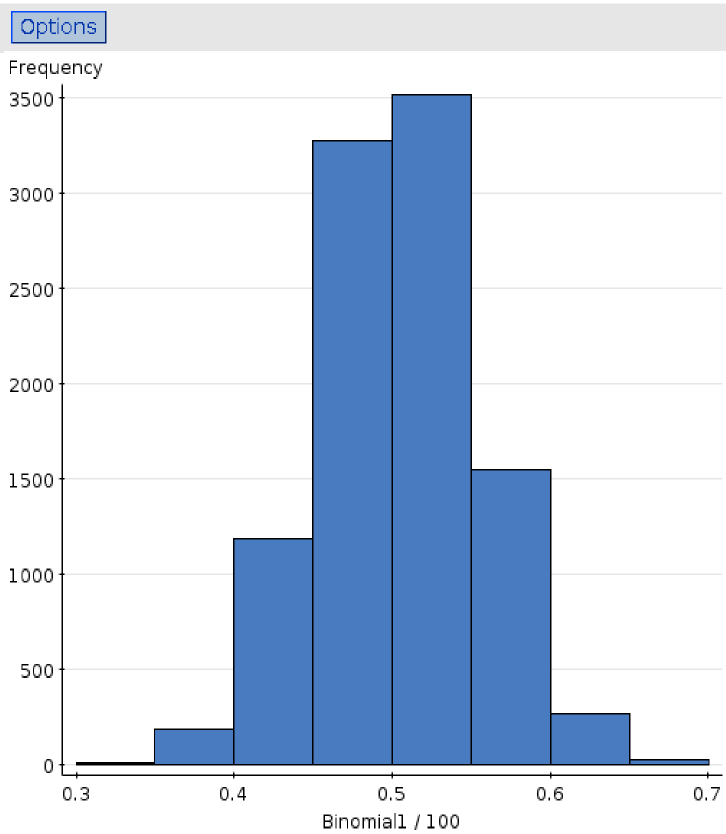
Either do it, or simulate with StatCrunch: binomial, n=100, p=0.5:

- number of heads not the same every time (sampling variability)
- usually between 40-60 heads
- usually *not* exactly 50 heads
- Do more simulations to get better idea.

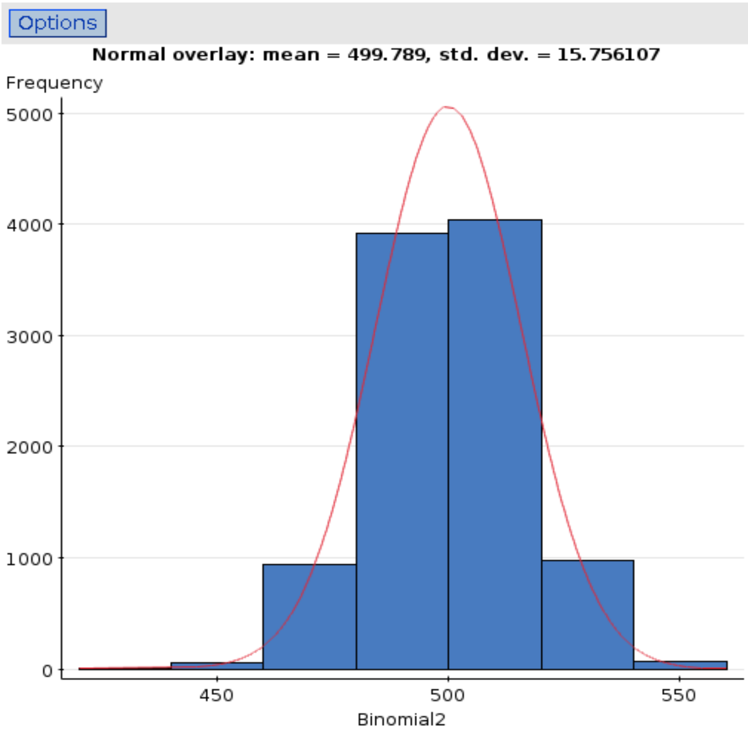
Number of heads in 100 tosses

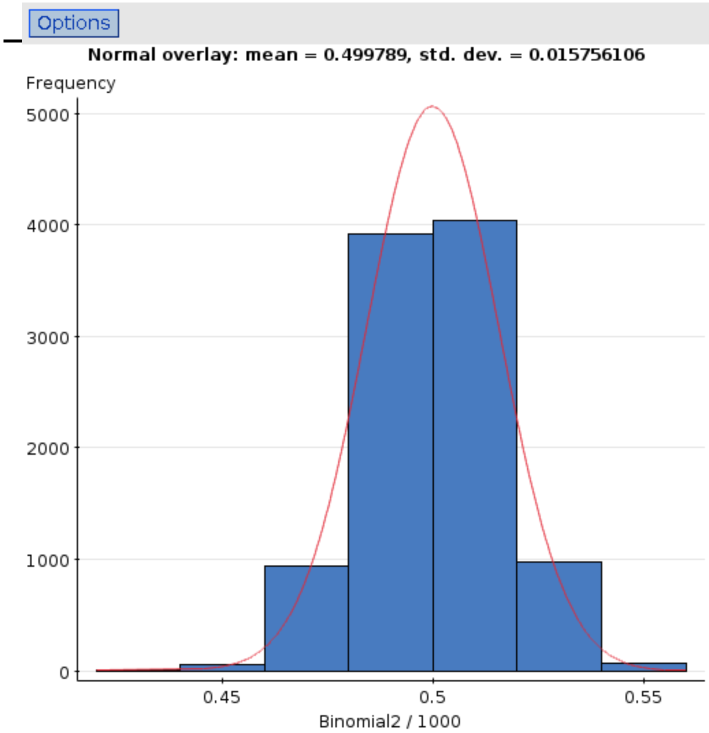
Proportion of heads





What about 1000 tosses instead of 100?





p

- proportion of heads likely *closer* to 0.5
- *number* of heads might be *further from* half #tosses
- shapes for n=100, n=1000 both normal

- **Normal approximation for counts and proportions**

Draw a SRS of size *n* from a large population having population *p* of success. Let X be the count of success in the sample and $\hat{p} = X/n$ the sample proportion of successes. When n is large, the sampling distributions of these statistics are approximately normal:

X is approx. $N(np, \sqrt{np(1-p)})$

$\hat{p}$ is approx. $N\left(p, \sqrt{\dfrac{p(1-p)}{n}}\right)$

Assumptions and Conditions

1. Randomization Condition: The sample should be a simple random sample of the population.

2. 10% Condition: If sampling has not been made with replacement, then the sample size, n, must be no larger than 10% of the population.

3. Success/Failure Condition: The sample size has to be big enough so that both np and nq are greater than 10.

Sampling distribution of a sample mean

If a population has the N(μ,σ), then the sample mean $\bar{X}$ of n independent observations has the N(μ,σ/$\sqrt{n}$)

The central limit theorem p485

Lottery:

Winnings	-1	2	10
Probability	0.9	0.09	0.01

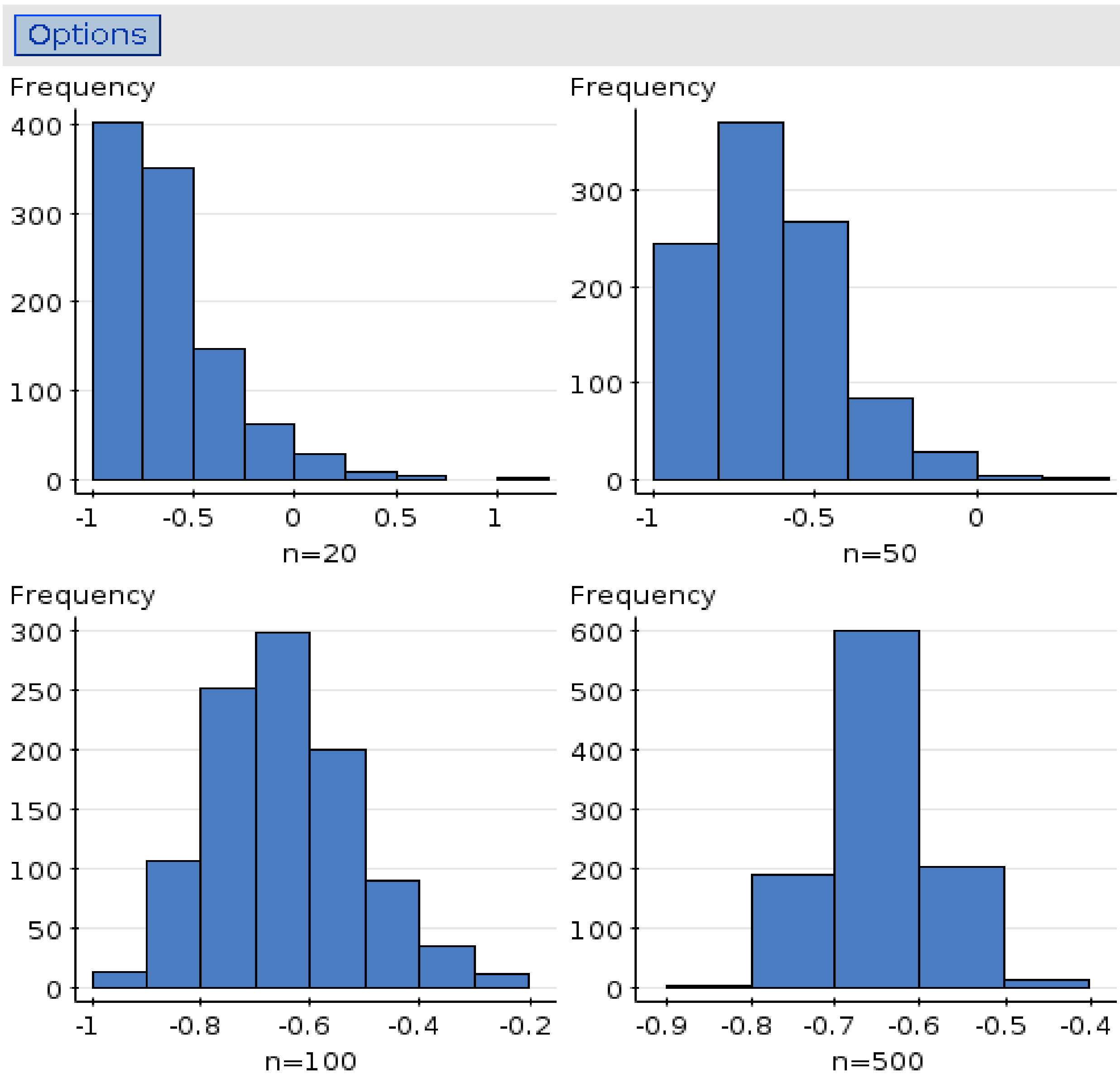
How much might you win per play if you play many times?

Mean winnings from 1 play is (-1)(0.9)+(2)(0.09)+(10)(0.01)=
-0.62 (population mean).

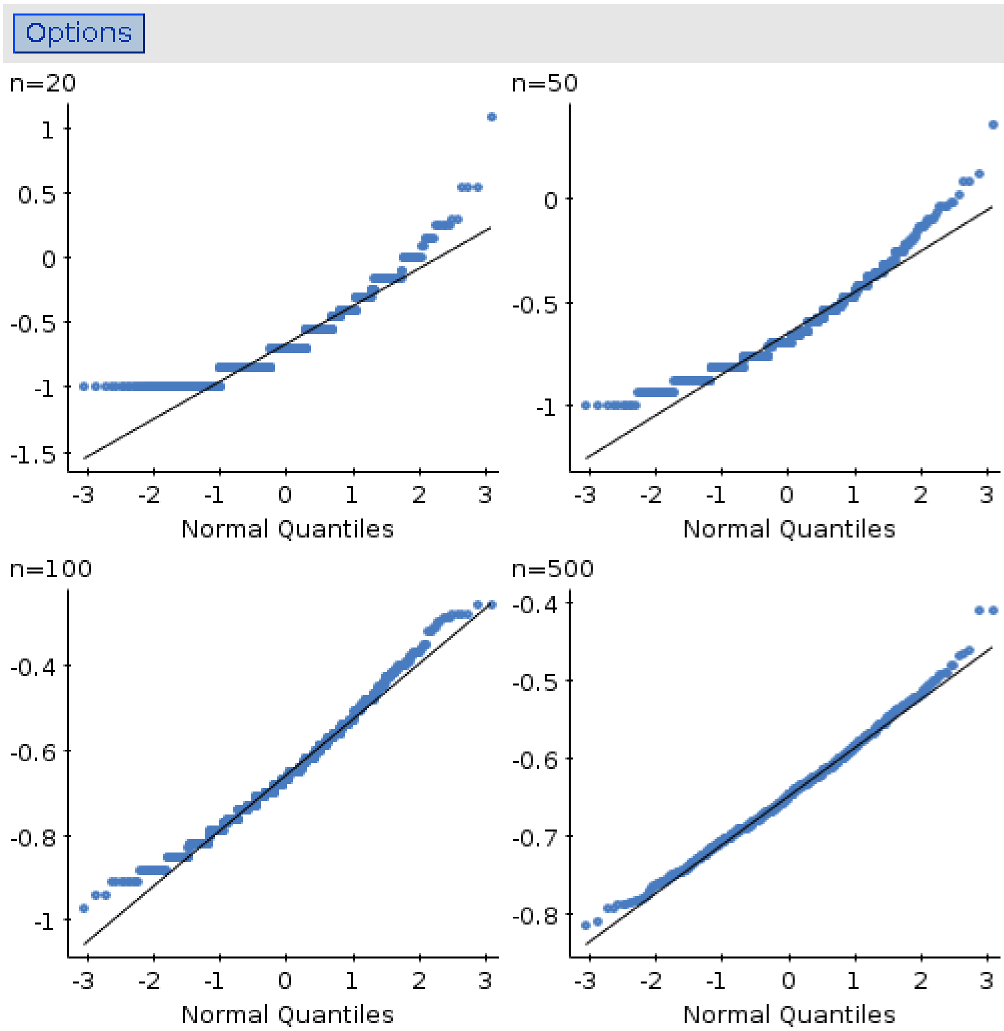
Law of large numbers: sample mean close to population mean for large enough sample. If you play 1000 times, you'll lose close to 0.62 per play.

What kind of sample means might you get for different sample sizes?

Sampling distributions of sample mean for various sample sizes



Normal quantile plots:



Skewed to right but progressively less so as n gets bigger: more and more normal.

Draw a SRS of size n from a population with mean μ and std dev. σ. When n is large, sampling distribution of a sample mean $\bar{X}$ is approximately normal with mean μ and std dev. $\sigma/\sqrt{n}$.

Note: The normal approximation for the sample proportion and counts is an important example of the central limit theorem.

Even in our very extreme population, began to work at about n=100.

Usually ok with *much smaller* n (eg. n=30 often big enough).

A sample of size n=25 is drawn from a population with mean 40 and SD 10. What is prob that sample mean will be between 36 and 44? (Assume Central Limit Theorem applies.)

Ex Suppose that the weights of airline passengers are known to have a distribution with a mean of 75kg and a std. dev. of 10kg. A certain plane has a passenger weight capacity of 7700kg. What is the probability that a flight of 100 passengers will exceed the capacity?

Ans: By CLT T~N(7500, sqrt(100x100)) P(T>7700)=P(Z>2)= 0.0228

Chapter 16: Confidence intervals for proportions p467

- Recall that the sampling distribution model of $\hat{p}$ is centred at *p*, with standard deviation $\sqrt{\frac{p(1-p)}{n}}$.
- Since we don't know *p*, we can't find the true standard deviation of the sampling distribution model, so we need to find the standard error:

$$SE(\hat{p})=\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Sampling distribution approx normal (when $np \geq 10$, and $n(1-p) \geq 10$)

Example Opinion poll with 1000 randomly sampled Canadians, 91% believe Canada's health care system better than US's.

$$SE(\hat{p})=\sqrt{\frac{(0.91)(1\text{-}0.91)}{1000}} = 0.0090.$$

Sampling distribution approx normal: $np \geq 10$, $n(1-p) \geq 10$

If we repeat sampling, about 95% of the time, sample proportion $\hat{p}$ should be inside

$$\left(p-2(0.0090), p+2(0.0090)\right) = p \pm 0.0180$$

that is, p and $\hat{p}$ should be less than 0.0180 apart.

In general, in repeated sampling, 95% of the intervals calculated using the formula $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ will contain *p*.

For any given sample the interval calculated using the formula $\hat{p} \pm z^{*}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ is called a **confidence interval.**

Note 2: **Margin of error** of the CI= $z^{*}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

The above CI can be written as $\hat{p} \pm z^{*} SE(\hat{p})$.

In the above example a 95% CI for the population proportion is

$0.91 \pm 1.96 \times \sqrt{\frac{0.91(1-1.91)}{1000}}$ = (0.891, 0.927)

Find z* for 90% interval:

- leftover is 10%=0.1000
- half that is 5%=0.0500
- Table: z=-1.64 or -1.65 has 0.0500 less
- z=1.64 or 1.65 has 0.0500 more (0.9500 less).
- so z*=1.64 or 1.65.

- Handy table:

Confidence level	z*
90%	1.645
95%	1.960
99%	2.576

What does “95% of the time” mean? *In 95% of all possible samples*. But different samples have different $\hat{p}$'s, and give different confidence intervals.

Eg. another sample, with n=1000, might have $\hat{p}=0.89$, giving 95% confidence interval for p of (0.870,0.910).

So *our confidence in procedure rather than an individual interval.*

Chapter 17, p 496: Testing Hypotheses about Proportions

A newsletter reported that 90% of adults drink milk. A survey in a certain region found that 652 of 750 randomly chosen adults (86.93%) drink milk. Is that evidence that the 90% figure is not accurate for this region?

Difference between 86.93 and 90, but might be chance.

One approach: confidence interval.

$\sqrt{\frac{\hat{p}\left(1-\hat{p}\right)}{n}}$ = 0.0123, so

95% CI is 0.845 to 0.893

99% CI is 0.838 to 0.901

so now what?

Hypothesis testing. Think about logic first by analogy.

Court of law

		Decision	
		Not guilty	Guilty
Truth	Innocent	Correct	Serious error
	Guilty	Error	Correct

- Truth (unknown)
- Decision (we hope reflects truth)
  - based on *evidence*: does it contradict accused being innocent?

- Null hypothesis H_0 is "presumption of innocence"

- Alternative hypothesis H_A is that H_0 is false. Need evidence (data) to be able to reject H_0 in favour of H_A .

Hypothesis testing

		Decision	
		fail to reject H_0	reject H_0
Truth	H_0 true	Correct	Type I error
	H_0 false	Type II error	Correct

Compare this with our example:

Court of law

		Decision	
		Not guilty	Guilty
Truth	Innocent	Correct	Serious error
	Guilty	Error	Correct

Example: A newsletter reported that 90% of adults drink milk. A survey in a certain region found that 652 of 750 randomly chosen adults (86.93%) drink milk. Is that evidence that the 90% figure is not accurate for this region?

Step 1 Set up the null and the alternative hypotheses:

- H_0: $p = 0.90$
- H_A: $p \neq 0.90$

Step 2: Calculate the test statistics:

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1 - p_0)}{n}}}$$

In our case, H_0: $p=0.90$ and $\hat{p}=652/750=0.8693$. If H_0 true, value of Z we might observe is approx standard normal.

$$z=\frac{\hat{p}-p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}=\frac{0.8693-0.9}{\sqrt{\frac{0.9(1-0.9)}{750}}}=-2.79$$

Step 3: P-value

P-value: The chance (the proportion) of getting a $\hat{p}$ as far or further from H_0 than the value observed.

The area under the std Normal curve below -2.79 is 0.0026.

Could have observed $\hat{p}$ *above* 0.90 too, so P-value twice this. i.e. P-value = 2 x 0.0026 = 0.0052

Step 4 Conclusion:

Reject the null hypothesis if the p-value is small.

How to decide whether P-value small enough to reject H_0?

Choose α ahead of time:

– if rejecting H_0 an important decision, choose small α (0.01)

"default" α = 0.05.

Reject H_0 if P-value less than the α you chose.

a value of $\hat{p}$ like the one we observed very unlikely *if* H_0: $p=0.90$ were *true* and so we reject H_0 .

One-sided and two-sided tests

Ex. Leroy, a starting player for a major college basketball team, made only 38.4% of his free throws last season. During the summer he worked on developing a softer shot in the hope of improving his free-throw accuracy. In the first eight games of this season Leroy made 25 free throws in 40 attempts. Let p be his probability of making each free throw he shoots this season.

(a) State the null hypothesis H0 that Leroy's free-throw probability has remained the same as last year and the alternative Ha that his work in the summer resulted in a higher probability of success.

(b) Calculate the z statistic for testing H0 versus Ha.

(c) Do you accept or reject H0 for $\alpha = 0{:}05$? Find the P-value.

(d) Give a 90% confidence interval for Leroy's free-throw success probability for the new season.

Are you convinced that he is now a better free-throw shooter than last season?

(a) H_0: p = 0.384 vs. H_a: p > 0.384. (b) $\hat{p} = \frac{25}{40}$ = 0.625, and z = $\frac{0.625 - 0.384}{\sqrt{(0.384)(0.616)/40}}$ = 3.13. (c) Reject H_0 (because the P-value < 0.05); P = 0.0009. (d) $SE_{\hat{p}}$ = $\sqrt{\hat{p}(1-\hat{p}/40}$ = $\sqrt{(0.625)(0.375)/40}$ = 0.0765, so the 90% confidence interval is 0.625 $\pm$ (1.645)(0.0765), or 0.4991 to 0.7509. Since this interval lies well above 0.384, there is strong evidence that Leroy has improved.