Study Questions #2 for STA 3000

These are for study only, not to hand in for credit

Q1: Show how the Poisson family of distributions can be expressed in exponential family form, and find natural parameters and natural sufficient statistics for this family. Derive formulas for the mean and variance of a Poisson random variable by taking derivatives of suitable functions of the normalizing constant with respect to the natural parameter.

Q2: Consider two models for non-negative integers X, Y, and Z that have positive real parameters α and β.

a) Suppose that X, Y, and Z are independent given α aned β, with $X|\alpha,\beta \sim \text{Poisson}(\alpha)$, $Y|\alpha,\beta \sim \text{Poisson}(\beta)$, and $Z|\alpha,\beta \sim \text{Poisson}(\sqrt{\alpha\beta})$. Show that this model has exponential family form, write it in non-degerate (ie, minimal) form, and say what the natural parameters and the natural stufficient statistics are for the form you wrote.

b) Suppose instead that X, Y, and Z are independent given α aned β, with $X|\alpha,\beta \sim \text{Poisson}(\alpha)$, $Y|\alpha,\beta \sim \text{Poisson}(\beta)$, and $Z|\alpha,\beta \sim \text{Poisson}(\alpha+\beta)$. Show that the minimal sufficient statistic for this model is not complete.