STA 3000, Fall 2009 — Assignment #3

Due December 11. Worth 8% of the course grade.

This assignment is to be done by each student individually. You may discuss it in general terms with other students, but the work you hand in should be your own. In particular, you should not leave any discussion of this assignment with any written notes or other recordings, nor receive any written or other material from anyone else by other means such as email.

Question 1: Consider a decision problem in which the data space, parameter space, and action space are all the set of real numbers. The distribution of the data, X, given the parameter, θ, is $N(\theta, 1)$. The loss function is $L(\theta, a) = (a - \theta)^2$.

Consider decision rules of the form $\delta_{a,b}(x) = ax + b$, where a and b can be any real numbers.

a) Find a simple formula for the risk function, $R(\theta, \delta_{a,b})$ for all a and b.

b) Which rules $\delta_{a,b}$ are dominated by some other rule of this form (ie, by $\delta_{a',b'}$ for some a' and b')?

c) For each of the rules of this form that are not dominated by another of this form, can you find a prior distribution for θ for which this rule is a formal Bayes rule?

d) Say what you can about the admissibility of each of the rules $\delta_{a,b}$.

Question 2: Consider a decision problem in which the data space, parameter space, and action space are all the set of positive real numbers. The distribution of the data, X, given the parameter, θ, is $U(0, 1 + \theta)$. We use a prior for which the probability density is proportional to $(1 + \theta)^{-r}$, where r is a fixed constant greater than one. We use the loss function $L(\theta, a) = (\theta - a)^2$.

a) For what values of r does a formal Bayes rule, δ_r, exist for all x? For those values of r, find a formal Bayes rule, and find a formula for the posterior risk, $r(\delta|x)$, of this rule.

b) For each formal Bayes rule, δ_r, that you found in (a), find the risk function, $R(\theta, \delta_r)$.

c) For what values of r is the formal Bayes rule, δ_r, that you found in (a) also a Bayes rule?

d) Say what you can about the admissibility of the formal Bayes rules you found in (a).