The University of Toronto

ACT451 Loss Models Test #2

You must show your steps or no point will be awarded

Name Solution

ID #__________

1. The ground-up loss X for a particular insured has a lognormal distribution with $\mu = 4$ and $\sigma = 2$. If an insurance policy on the loss has an ordinary deductible of 200 and a maximum covered loss of 1500.

a) 5 points Determine the expected cost per loss for this policy.

$X \sim \text{lognormal}(\mu=4, \sigma=2)$

$E(Y_L) = E(X\wedge u) - E(X\wedge d) = E(X\wedge 1500) - E(X\wedge 200)$

$E(X\wedge 1500) = e^{4+\frac{1}{2}(2)^2}\Phi\left(\frac{\ln 1500-4-4}{2}\right) + 1500\left[1-\Phi\left(\frac{\ln 1500-4}{2}\right)\right]$

$= e^6\Phi(-0.343) + 1500[1-\Phi(1.657)] = e^6(1-0.6331) + 1500(1-0.9515)$

$= 403.429(0.3669) + 1500(0.0485) = 220.7681$

$E(X\wedge 200) = e^6\Phi\left(\frac{\ln 200-4-4}{2}\right) + 200\left[1-\Phi\left(\frac{\ln 200-4}{2}\right)\right]$

$= e^6\Phi(-1.351) + 200[1-\Phi(0.649)] = e^6(1-0.9115) + 200(1-0.7422)$

$= 403.429(0.0885) + 200(0.2578) = 87.2635$

$\Rightarrow E(Y_L) = 220.7681 - 87.2635 = 133.5046$ //

b) 5 points Determine the expected cost per payment for this policy.

$$E(Y_P) = \frac{E(Y_L)}{P(Y_L>0)}$$

$P(Y_L>0) = 1-F_X(d) = 1-F_X(200) = 1-\Phi\left(\frac{\ln 200-4}{2}\right) = 1-\Phi(0.649)$

$= 1-0.7422 = 0.2578$

From part a), $E(Y_L) = 133.5046$

$$\Rightarrow E(Y_P) = \frac{133.5046}{0.2578} = 517.8611 \text{ //}$$

c) 5 points Determine the second moment of cost per loss.

$$E(Y_L^2) = E(X\wedge u)^2 - E(X\wedge d)^2 - 2d\left[E(X\wedge u) - E(X\wedge d)\right]$$
$$= E(X\wedge 1500)^2 - E(X\wedge 200)^2 - 2(200)E(Y_L)$$

$$E(X\wedge 1500)^2 = e^{2(4)+(2^2)(2)^2(\frac{1}{2})}\Phi\left(\frac{\ln 1500 - 4 - 2(2)^2}{2}\right) + (1500)^2\left[1 - \Phi\left(\frac{\ln 1500 - 4}{2}\right)\right]$$
$$= e^{16}\Phi(-2.343) + (1500)^2\left[1 - \Phi(1.657)\right]$$
$$= e^{16}(1 - 0.9904) + (1500)^2(1 - 0.9515)$$
$$= e^{16}(0.0096) + (1500)^2(0.0485) = 194431.661.$$

$$E(X\wedge 200)^2 = e^{16}\Phi\left(\frac{\ln 200 - 4 - 2(2)^2}{2}\right) + (200)^2\left[1 - \Phi\left(\frac{\ln 200 - 4}{2}\right)\right]$$
$$= e^{16}\Phi(-3.351) + (200)^2\left[1 - \Phi(0.649)\right]$$
$$= e^{16}(1 - 0.9996) + (200)^2(1 - 0.7422) = 13866.4442$$

From part a), $E(Y_L) = 133.5046$

$$\Rightarrow E(Y_L^2) = 194431.661 - 13866.4442 - 2(200)(133.5046)$$
$$= 127163.3768 \quad //$$

d) **5 points** A gross premium for this policy is charged so that the probability that the payment by its issuer would be less than the premium is 0.8. Determine the exact premium. (payment includes 0).

let Q be the premium.

$P(Q > Y_L) = 0.8 \iff P(Q > X - 200) = 0.8 \iff P(Q + 200 > X) = 0.8$

So $\Phi\left(\frac{\ln(Q+200) - 4}{2}\right) = 0.8$

$\Rightarrow \ln(Q+200) = 4 + 2 \cdot \Phi^{-1}(0.8) = 4 + 2(0.842) = 5.684$

$\Rightarrow Q = e^{5.684} - 200 = 94.124$ //

2. An insurance company sells a property insurance policy with an ordinary deductible of 200 and policy limit of 2000, and a coinsurance factor of 80%. The underlying ground-up loss of this policy follows a 2-parameter Pareto with $\theta = 3,000$ and $\alpha = 4$.

a) 5 points Determine the expected cost per loss.

$X \sim \text{Pareto}(\alpha=4,\ \theta=3000)$

$d = 200$.

$0.8(u-d)=2000 \Rightarrow u=2700$.

$$E(Y_L) = 0.8[E(X\wedge u)-E(X\wedge d)] = 0.8[E(X\wedge 2700)-E(X\wedge 200)]$$

$$= 0.8\left[\frac{3000}{4-1}\left(1-\left(\frac{3000}{2700+3000}\right)^{(4-1)}\right) - \frac{3000}{4-1}\left(1-\left(\frac{3000}{200+3000}\right)^{(4-1)}\right)\right]$$

$$= 0.8[1000(1-0.14579)-1000(1-0.82397)]$$

$$= 0.8(854.21-176.03)$$

$$= 0.8(678.18)$$

$$= 542.544$$

b) 5 points Determine the second moment of the cost per payment.

$$E(Y_P)^2 = (0.8)^2\frac{E(Y_L)^2}{P(Y_L>0)} = (0.8)^2\left[\frac{E(X\wedge 2700)^2-E(X\wedge 200)^2-2(200)[E(X\wedge 2700)-E(X\wedge 200)]}{1-F_X(200)}\right]$$

$$1-F_X(200) = \left(\frac{\theta}{x+\theta}\right)^\alpha = \left(\frac{3000}{3200}\right)^4 = 0.77248.$$

$$E(X\wedge u)^2 = 2\theta\left[\frac{\theta}{\alpha-2}\left(1-\left(\frac{\theta}{\theta+u}\right)^{\alpha-2}\right) - \frac{\theta}{\alpha-1}\left(1-\left(\frac{\theta}{\theta+u}\right)^{\alpha-1}\right)\right]$$

$$E(X\wedge 2700)^2 = 2(3000)[1500(1-0.277)-1000(1-0.1458)]$$
$$= 6000[1500(0.723)-1000(0.8542)] = 1381800.$$

$$E(X\wedge 200)^2 = 2(3000)[1500(1-0.8789)-1000(1-0.824)]$$
$$= 6000[1500(0.1211)-1000(0.176)] = 33900$$

$$E(X\wedge 2700)-E(X\wedge 200) = \frac{542.544}{0.8} = 678.18.$$

$$\Rightarrow E(Y_P)^2 = (0.64)\cdot\left(\frac{1381800-33900-2(200)(678.18)}{0.77248}\right) = 891986.744$$

3. 5 points N is a random variable from the (a,b,1) class. You are given:

$$P\{N=0\}=0.25, P\{N=1\}=0.23798, P\{N=2\}=0.06545, P\{N=3\}=0.01200.$$

Identify this distribution and its parameter values.

$$\frac{P(N=2)}{P(N=1)} = 0.275023111 = a + \frac{b}{2}$$

$$\frac{P(N=3)}{P(N=2)} = 0.183346065 = a + \frac{b}{3}$$

Solve to get,

$a \doteq 0$ and $b = 0.55$

$\therefore N \sim$ zero-modified poisson with $\lambda = 0.55$

$P(N=0) = p_0^M = 0.25$. //

4. The aggregagte claims amount S has a compound negative binomial distribution with negative binomial parameters $\beta = 0.5$ and $r = 3$. The individual claim amounts have a gamma distribution with $\theta = 30$ and $\alpha = 2$.

a) 5 points Determine the expected aggregate claims amount.

$N \sim NB(r=3, \beta=0.5)$

$X \sim \text{Gamma}(\alpha=2, \theta=30)$.

$E(S) = E(N) \cdot E(X)$

$E(N) = r \cdot \beta = (3)(0.5) = 1.5$

$E(X) = \alpha \cdot \theta = (2)(30) = 60$.

$\Rightarrow E(S) = (1.5)(60) = 90$ //.

b) 5 points Identify the MGF of S.

$M_S(t) = P_N(M_X(t))$.

$P_N(t) = [1-\beta(t-1)]^{-r}$

$M_X(t) = (1-\theta t)^{-\alpha}$.

$$\Rightarrow M_S(t) = [1-\beta((1-\theta t)^{-\alpha}-1)]^{-r} = [1-\tfrac{1}{2}((1-30t)^{-2}-1)]^{-3}$$

$$= \left[\frac{(1-30t)^2}{(1-30t)^2-\beta(1-(1-30t)^2)}\right]^3 = \left[\frac{(1-30t)^2}{1.5(1-30t)^2-0.5}\right]^3$$

$$= \frac{(1-30t)^5}{(1.5(1-30t)^2-0.5)^3}$$

//.