7. You are to approximate the aggregate claims distribution of the individual model by a compound Poisson model. The two standard approximations described in $\underline{Actuarial\ Mathematics}$ are considered:
(1) the expected number of claims is the same as in the individual model;
(2) the probability of no claims occurring is the same as the individual model.
What is the correct ranking of $Var[S]$, $Var[S_1]$, $Var[S_2]$.

A) $Var[S] < Var[S_1] < Var[S_2]$ B) $Var[S] < Var[S_2] < Var[S_1]$
C) $Var[S_1] < Var[S] < Var[S_2]$ D) $Var[S_1] < Var[S_2] < Var[S]$
E) $Var[S_2] < Var[S_1] < Var[S]$

7. Solution $Var[S] = \Sigma[q_j(1-q_j)\mu_j^2 + q_j\sigma_j^2]$, $Var[S_1] = \Sigma q_j(\mu_j^2 + q_j\sigma_j^2)$
$Var[S_2] = \Sigma - log_e(1-q_j)(\mu_j^2 + q_j\sigma_j^2)$.

Since $q_j(1-q_j) < q_j < q_j + \frac{1}{2}q_j^2 + \frac{1}{3}q_j^3 + \cdots = -log_e(1-q_j)$, it follows that
$Var[S] < Var[S_1] < Var[S_2]$. Answer: A.

7a. Consider a portfolio of 100 lives:

Benefit	Number of Lives	Probability of a claim, q
1	15	.02
1	45	.04
4	30	.02
4	60	.04

You are to approximate the aggregate claims distribution of the individual model by a compound Poisson model. The two standard approximations described in $\underline{Actuarial\ Mathematics}$ are considered:
(1) the expected number of claims is the same as in the individual model;
(2) the probability of no claims occurring is the same as the individual model.
The insurer will charge a premium of $P = E[S] + \sqrt{Var[S]}$. What is P_1/P_2 , where P_1 and P_2 are based on approximations 1 and 2, respectively.

A) .975 B) .985 C) .995 D) 1.005 E) 1.015

7a. Solution For approximation 1, $\lambda = 15 \cdot (.02) + 45 \cdot (.04) + 30 \cdot (.02) + 60 \cdot (.04) = 5.10$,
$E[S_1] = \Sigma\, q_j\mu_j = 14.10$, and, since $\sigma_j^2 = 0$ for all j, $Var[S_1] = \Sigma\, q_j(\mu_j^2 + \sigma_j^2) = \Sigma\, q_j\mu_j^2 = 50.10$.
For approximation 2,
$\lambda' = -15log_e(1-.02) - 45log_e(1-.04) - 30log_e(1-.02) - 60log_e(1-.02) = 5.1954$,
$E[S_2] = \Sigma\ - log_e(1-q_j)\mu_j = 14.3616$, and again, since $\sigma_j^2 = 0$ for all j,
$Var[S_2] = \Sigma\ - log_e(1-q_j)\mu_j^2 = 51.0264$.
$P_1 = 14.10 + \sqrt{50.10} = 21.178$ and $P_2 = 14.3616 + \sqrt{51.0264} = 21.5049$,
so that $P_1/P_2 = .985$. Answer: B.

17. A portfolio of insurance policies is summarized as follows :

Number of Policies	Probability of Claim	Distribution of Claim Amount Mean	Variance
20	.01	2	1
n	.02	1	c
25	.03	2	2

Using the individual risk model, the variance of aggregate claims is 6.4932 . A compound Poisson approximation is applied, in which the expected number of claims in the approximation is set equal to the expected number of claims in the individual model. The variance of aggregate claims in the compound Poisson approximation is 6.600 . What is c ?

A) .5 B) 1.0 C) 1.5 D) 2.0 E) 2.5

17. Solution $Var[S] = \Sigma q_j(1-q_j)\mu_j^2 + q_j\sigma_j^2 \ = \ 5.402 + .0196n + .02cn \ = \ 6.4932$,
$Var[S_1] = \Sigma q_j(\mu_j^2 + \sigma_j^2) \ = \ 5.50 + .02n + .02cn \ = \ 6.600$.
Subtracting the first equation from the second equation results in $.098 + .0004n = .1068$, so that $n = 22$, and then $c = 1.5$. Answer: C.

9. A compound claim distribution is analyzed and it is decided to model the distribution of S by using the geometric distribution for N (claim number) and the gamma distribution for X (claim amount). With this model it is found that $E[S] = 16$, $Var[S] = 328$ and $Var[N] = 20$. An alternative model is proposed in which the distribution of N is Poisson with the same mean as the geometric distribution for N in the first model. The claim amount distribution in the new model is the same gamma distribution as in the old model, and the mean aggregate claims in the new model is the same as in the old model (16). What is the variance of the aggregate claims random variable in the new model?

A) 18 B) 36 C) 54 D) 72 E) 90

9. Solution With a geometric distribution for N, $Var[N] = \frac{1-p}{p^2} = 20 \ \rightarrow \ 20p^2 + p - 1 = 0 \ \rightarrow \ p = .2$ (ignore the negative root $-.25$). Then $E[N] = \frac{1-p}{p} = 4$. Since $E[S] = E[N] \cdot E[X]$ it follows from the parameters in the old model that $E[X] = 4$. Since $Var[S] = E[N] \cdot Var[X] + Var[N] \cdot (E[X])^2$, it follows from the parameters of the old model that $Var[X] = 2$. Then, $p_2 = Var[X] + p_1^2 = 18$, and $Var[S] = \lambda \cdot p_2 = 72$. Answer: D.

8. An insurer has three independent blocks of business, each of which has a compound Poisson distribution. The expected numbers of claims per period for the three blocks of business are 2 , 3 and 3, respectively. The claim amount distributions are discrete for each of the three blocks of business and are summarized as follows:

x	$P_1(x)$	$P_2(x)$	$P_3(x)$
1	0	.2	.2
2	.4	.5	.2
3	.8	.8	.6
4	1.0	1.0	1.0

$P(x)$ denotes the claim amount distribution for the combined blocks of business. What is $P^{*3}(5)$?

A) .0500 B) .0625 C) .0750 D) .0875 E) 1.000

8. Solution With $\lambda_1 = 2$, $\lambda_2 = 3$, $\lambda_3 = 3$ and $\lambda = 8$, and using $P(x) = \sum_{i=1}^{3} \frac{\lambda_i}{\lambda} \cdot P_i(x)$, we see that

The distribution of X for the combined blocks of business is

x	$P(x)$	$p(x)$

1	.15	.15
2	.3625	.2125
3	.725	.3625
4	1.0	.275

Applying the method of convolution we have

x	$P(x)$	$P^{*2}(x)$	$P^{*3}(x)$
1	.15	0	0
2	.3625	.0225	0
3	.725	.08625	.003375
4	1.0	.240156	.017719
5	1.0	.476719	.062508

Answer: B.