

Two Most Important Risk Measures

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- **Value at Risk (VaR)**

Definition:

Let $X > 0$ be a loss random variable with distribution function $F_X(x)$ or density function $f_X(x)$.

The Value at Risk at confidence level p is defined as

$$VaR_p = \inf\{x; F_X(x) \geq p\}.$$

In other words, VaR_p is the 100 p -th percentile of the distribution $F_X(x)$.

- **Conditional Value at Risk (CVaR) or Conditional Tail Expectation (CTE)**

Definition:

The Conditional Value at Risk at confidence level p is defined as

$$CVaR_p = E\{X \mid X > VaR_p\}.$$

CVaR is the term commonly used in Finance. It is also called Expected Shortfall and Conditional Tail Expectation (by North American actuaries) with notation $CTE(100p)$.

$CVaR_p$ is always greater than VaR_p and the difference $CVaR_p - VaR_p$ is the mean residual lifetime (MRL) in reliability theory.

Both measures are highly dependent of the loss distribution and its tail behaviour in particular.

Roles of VaR and CTE in Practice

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- VaR is a probability of ruin measure;
CTE is a cost of ruin measure.
 p is viewed as the level of security to target.
- Both are used for calculation of the required capitals for liabilities/losses over a one-year period.
- Apply to both individual risks and aggregate risks.
- Both are used as a base for stress/capital adequacy/solvency tests.
- Can be used for risk-based pricing.
- May have a role in mergers and acquisitions.

credit rating	BBB	A	AA	AAA
p	0.90	0.95	0.975	0.99