

Valuation of Equity-Indexed Annuities under Stochastic Interest Rates

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Abstract

In this paper, we consider the pricing of equity-indexed annuities (EIAs). Traditionally the values of the guarantees embedded in these contracts are priced by modeling the underlying index fund while keeping the interest rates constant. The assumption of constant interest rates becomes unrealistic in pricing and hedging the EIAs since the embedded guarantees are often of much longer maturity. To solve this problem, we propose an economic model which has the flexibility of modeling the underlying index fund as well as the interest rates. Some popular EIAs are illustrated to assess the implication of the proposed model.

1 Introduction

An equity-indexed annuity (EIA) is a fixed annuity that earns a minimum rate of interest and offers a potential gain that is tied to the performance of an equity index. The first EIA product was introduced in 1995 by Keyport Life Insurance Co. Since then EIAs have enjoyed some popularity in both U.S. and Canada. The sales of EIAs had increased from 1.5 billion in 1996 to 5.2 billion in 1999. For years 2000 and 2001, the sales of EIAs remain stable at 6 billion per year. Despite the downturn of the stock market, there seems to have a renewed public interest in EIAs recently, as evidenced by the record high sales of near 12 billion in 2002. See Marrion's reports (2000, 2001, 2002) and <http://www.indexannuity.org> for details.

EIAs appeal to investors because they not only offer some of the benefits underlying conventional annuities but also offer participation in the equity market while limiting downside risk of the equity market. A typical EIA guarantees a minimum return (normally 3%) on a portion of the initial amount invested. In addition to this minimum guarantee, the annuitant receives some participation in the appreciation of a pre-determined equity index such as S&P 500. The indexing feature extends over a fixed term, typically ranging from one to ten years. There are several indexing methods for EIAs. In the order of decreasing sales volumes, they are Annual Reset, Annual Yield Spread, Point-to-Point, High-Water Mark, and Term Yield Spread. The index growth on an EIA with Annual Reset option is measured each year by comparing the index level at the beginning and the end of the year. The index growth with Point-to-Point indexing is based on the growth between two time points. As in the Annual Reset method, the Annual Yield Spread method resets the index growth annually but a yield spread is deducted from the equity index. The index growth with High-Water Mark feature (also called term High-Point or Point-to-Point with discrete lookback) is calculated to the highest index anniversary value over the entire term of the annuity. The Term Yield Spread method is similar to the Annual Yield Spread method except that a yield spread is deducted for the entire term of the EIA. In addition to the above methods, an averaging scheme is often used to calculate the index growth in order to reduce the costs of the guarantees and to be partly immunized from the market volatility.

There have been several researches on this subject. See the monograph by Hardy (2003) (and also Tiong 2000) for comprehensive discussion on this topic. In general, it is often assumed that the equity index and interest rates are within a Black-Scholes framework; i.e. the equity index follows a lognormal process with constant interest rates. In this research, we consider a more general economic model assuming that the interest rates are stochastic and follow a diffusion process. Our objective is to conduct a detailed numerical analysis on EIAs currently sold in the North American market and to examine the impact of the equity index and interest rates on the evaluation of these EIAs. This paper is organized as follows. We present the joint economic model and valuation formulas in Sections 2, 3 and 4. In Section 5, we examine the implication of the proposed model to the conventional model

by conducting a detailed analysis on the most popular types of embedded equity guarantees. Section 6 concludes the paper.

2 Economic Model Selections

Two crucial economic factors in the valuation and hedging of an EIA are the term structure of interest rates and the level of the equity index. The research to-date has primarily focused on modeling just one of the key variables. For example, in the Black-Scholes framework, the index is stochastic while other variables such as volatility or interest rates are constant. While these assumptions might be adequate for most options offered by the exchanges and banks, it is some time undesirable to extrapolate that these assumptions are also applicable to the guarantees embedded in EIAs. Most of the options offered by the exchanges and banks typically are short-dated with maturity less than one year, and hence a Black-Scholes framework would provide a reasonable approximation for pricing purposes. In contrast, the embedded guarantees associated with EIAs have maturities ranging from 1 to 10 years. It is therefore unreasonable to assume that the interest rates would remain level for such a long duration.

In this section we jointly model the term structure of interest rates and the equity index using stochastic differential equations. The basic model consists of a stochastic differential equation for the short term interest rate and a stochastic differential equation for the equity index. We wish that (1) the short rate process reproduces the current yield curve as well as bond prices at different maturities; (2) the model incorporates correlation between the interest rate and the equity index, which is accomplished by explicitly introducing correlation between the diffusion processes; and (3) the model is mathematically or computationally tractable in the sense it can be implemented using sophisticated mathematical or numerical tools such as the Monte Carlo simulation.

We now describe the interest rate and equity processes. Let $r(t)$ be the short rate at time t for $0 \leq t \leq \mathcal{T}$, where \mathcal{T} is the time-to-maturity of an EIA. We assume that the short rate process $\{r(t)\}, 0 \leq t \leq \mathcal{T}$, satisfies the following stochastic differential equation

$$dr(t) = \mu_r(t, r)dt + \sigma_r(t, r)dW_r(t), \quad (2.1)$$

where the drift $\mu_r(t, r) = \kappa[\theta(t) - r]$ for some deterministic function $\theta(t)$, the volatility of the short rate process $\sigma_r(t, r) = \sigma_r$, and $\{W_r(t)\}$ is a standard Brownian motion. Let $S(t)$ be the equity index level at time t which is governed by a stochastic differential equation of the form:

$$dS(t) = \mu_S(t)S(t)dt + \sigma_S(t)S(t)dW_S(t). \quad (2.2)$$

Here $\mu_S(t)$ represents the instantaneous rate of return of the index at time t and $\sigma_S(t) > 0$ is the volatility of the index at time t both of which are deterministic, and $\{W_S(t)\}$ is a standard Brownian motion that is correlated with $\{W_r(t)\}$ with correlation coefficient ρ ; i.e.

$$\text{corr}(W_r(t), W_S(t)) = \rho.$$

For valuation purposes, the underlying probability measure associated with these processes is assumed to be the risk-neutral measure or Q -measure under which the present value process

$$V(t) = e^{-\int_0^t r(u)du} S(t) \quad (2.3)$$

is a martingale.¹ Hence we have $\mu_S(t) = r(t)$. We now make some remarks. The short rate model (2.1) is often referred to as the extended Vasicek model (Vasicek, 1977) or the Hull and White model (Hull and White, 1990). It can be shown that a closed-form solution for $r(t)$ exists and is given by

$$r(t) = r(0)e^{-\kappa t} + \kappa \int_0^t e^{-\kappa(t-u)} \theta(u) du + \sigma_r \int_0^t e^{-\kappa(t-u)} dW_r(u). \quad (2.4)$$

As a result, the price of default-free discount bonds with all possible maturities may be derived explicitly, since for a discount bond maturing at time t its price is given by $E \left(e^{-\int_0^t r(s)ds} \right)$. This thus allows the current zero rate curve to be derived. Hence we could choose a proper function form of $\theta(t)$ such that the model zero rate curve coincides with the market zero rate curve. However, in this paper it is assumed that $\theta(t)$ remains a constant i.e. $\theta(t) = \theta$, for all t , for illustration purposes, as considered in Chan et. al. (1992), Ait-Sahalia (1996), and Duan and Simanato (1999). Implementation of the above model with a deterministic function can be accommodated with minor modifications. As we start with the risk-neutral probability measure for the model, the parameter θ is of the form $\theta = \theta_0 + \sigma\lambda/\kappa$, where θ_0 represents the long-term average of the short rate $r(t)$ and λ is the market price of risk under the physical measure. It should be pointed out that the model could generate negative short rates since $r(t)$ is a normal random variable for each t . However, in most practical applications the probability of having a negative interest rate is very small and hence it still serves as a reasonable model due to its tractability. For further information on term structure interest rate models and their parameter estimation issues, see James and Webber (2000).

Let $C(s)$ be the payoff in year s for an initial one monetary unit investment in an EIA. By definition, the payoff $C(s)$ is a contingent claim as it depends on the values of the index and the short rates up to time s . Thus, in the absence of mortality risk, the time- t price of $C(s)$, $t < s$, is given by

$$\Pi(t, s) = E \left[e^{-\int_t^s r(u)du} C(s) \mid \mathcal{F}_t \right], \quad (2.5)$$

where \mathcal{F}_t is the time- t information structure generated by $\{r(t)\}$ and $\{S(t)\}$ (Harrison and Pliska, 1981). Intuitively, the time- t price is simply the expected (short rate) discounted payoff under the risk-neutral probability measure Q . In a special case where $t = 0$, the time-0 price becomes

$$\Pi(0, s) = E \left[e^{-\int_0^s r(u)du} C(s) \right].$$

¹A stochastic process $\{V(t)\}$ is a martingale if, for any $t < s$, $E[V(s) \mid \mathcal{F}_t] = V(t)$. See Øksendal (1998), p. 31.

It is easy to see that using the Law of Iterated Expectations, the above expression is equivalent to

$$\Pi(0, s) = \mathbb{E} \left[e^{-\int_0^t r(u)du} \Pi(t, s) \right].$$

Furthermore, since $\mathbb{E} \left[e^{-\int_0^s r(u)du} C(s) \mid \mathcal{F}_t \right]$ is a martingale, $e^{-\int_0^t r(u)du} \Pi(t, s)$ is also a martingale. It follows from $\sigma_S(t) > 0$ and the Martingale Representation Theorem (see for example Øksendal, 1998, p. 53) that

$$d[e^{-\int_0^t r(u)du} \Pi(t, s)] = \delta(t, s) dV(t), \quad (2.6)$$

for a (predictable) stochastic process $\delta(t, s)$. Here the present value of the index, $V(t)$, is given in (2.3). Applying the multiplication rule of the Itô calculus, we obtain

$$d\Pi(t, s) = r(t)\phi(t, s)dt + \delta(t, s)dS(t), \quad (2.7)$$

with

$$\phi(t, s) = \Pi(t, s) - \delta(t, s)S(t). \quad (2.8)$$

Thus, we may replicate the payoff $C(s)$ using a self-financing trading strategy $\{\phi(t, s), \delta(t, s)\}$, where the replicating portfolio consists of the amount $\phi(t, s)$ in the money market account and $\delta(t, s)$ units of the index fund at time t .

3 Incorporating Mortality Risk

In this section, we incorporate mortality risk into our analysis. We will discuss how to evaluate an EIA using the economic model in Section 2 while taking into account the mortality risk. As we have seen in the last section, a fundamental idea of pricing a contingent payoff in the financial market using the risk-neutral probability measure is that one can perfectly replicate the payoff of the contingent claim by rebalancing a portfolio consisting of the underlying risky asset(s) and the money market account with a self-financing strategy. A necessary condition for this pricing principle to hold is that all financial assets involved must be tradable over time. When a payoff depends not only on risky assets but also contingents on the mortality of the holder, this condition is violated and the pricing principle discussed in the last section is no longer applicable. Consequently, a perfect hedging using a self-financing trading strategy becomes impossible. A possible solution to overcome this is to consider the risk-minimizing hedging strategy proposed by Schweizer (1994) (see also Møller (1998, 2001a, 2001b)). We now describe the underlying principle associated with this method.

We begin by introducing necessary actuarial symbols. As in Bowers et al. (1997), let (x) be an annuitant who purchases an EIA at age x and $T(x)$ be the future lifetime of (x) . Also let ${}_t p_x$ and ${}_t q_x$ be, respectively, the probability of survival and death; i.e. ${}_t p_x = P(T(x) > t)$

and ${}_tq_x = 1 - {}_tp_x$ and with the convention that ${}_1q_x = q_x$. Furthermore, the force of mortality is defined as $\mu(x+t) = -\frac{d}{dt} \ln {}_tp_x$.

We assume that the future lifetime $T(x)$ is stochastically independent of the Brownian motions $W_r(t)$ and $W_S(t)$. Consequently, $T(x)$ is also stochastically independent of $r(t)$ and $S(t)$. Intuitively, this means that the event of death is independent of the interest rates and the equity index and vice versa, and hence the mortality risk can be diversified by selling policies of the same kind to enough people.²

Let $C(t)$ be the payoff at time t of an EIA. For simplicity, we assume that there is no surrender and death benefits are paid at the end of year of death. The present value of the payoff, C , can be defined as

$$C = \sum_{j=0}^{\mathcal{T}-1} e^{-\int_0^{j+1} r(u)du} C(j+1)I(j < T(x) \leq j+1) + e^{-\int_0^{\mathcal{T}} r(u)du} C(\mathcal{T})I(T(x) > \mathcal{T}), \quad (3.1)$$

where \mathcal{T} is the time of maturity and $I(A)$ is the indicator function of event A ; i.e., $I(A) = 1$ if A is true, otherwise $I(A) = 0$. Consider now a portfolio that consists of the index fund and a money market account that earns interest at rate $r(t)$ at time t . It is required that this portfolio will meet the liability obligation given above. Let $\delta(t)$ be the number of units of the index fund at time t and $\phi(t)$ be the amount in the money market account at time t . Thus the value of this portfolio, $PO(t)$, at time t is given by

$$PO(t) = \phi(t) + \delta(t)S(t),$$

and its present value, $PV(t)$,

$$PV(t) = e^{-\int_0^t r(u)du} PO(t) = e^{-\int_0^t r(u)du} \phi(t) + \delta(t)V(t).$$

Note that we have the condition $PV(\mathcal{T}) = C$ to ensure meeting the liability obligation. The present value of the accumulated costs up to time t for maintaining the portfolio from the above trading strategy is thus

$$CO(t) = PV(t) - \int_0^t \delta(u)dV(u), \quad (3.2)$$

²We remark that we compromise the mathematical rigor in the above discussion in order to present the intuitive idea on how to integrate the mortality risk into the model. In the above, we need to expand the information structure $\{\mathcal{F}_t\}$ associated with $r(t)$ and $S(t)$ and the risk neutral probability measure Q to include the mortality law. The new information structure $\{\tilde{\mathcal{F}}_t\}$ is such that for each t , $\tilde{\mathcal{F}}_t$ is the smallest information structure containing \mathcal{F}_t and the information structure generated by $T(x)$ while maintaining the independence between $\{\mathcal{F}_t\}$ and the information structure generated by $T(x)$. We denote this the new risk-neutral probability measure \tilde{Q} . It should be noted that there are many ways to expand the risk-neutral probability measure Q to all events in $\{\tilde{\mathcal{F}}_t\}$ and the one we adopt is called the minimal martingale measure by Schweizer (1995). Furthermore, the mortality law remains the same. For detailed description, see Møller (1998), Section 2.3. For notational simplicity, we again denote $\{\tilde{\mathcal{F}}_t\}$ and \tilde{Q} by $\{\mathcal{F}_t\}$ and Q , respectively. It should also be pointed out that the independence between investment risk and mortality risk in the physical measure does not necessarily imply their independence in the risk-neutral measure.

where $\int_0^t \delta(u)dV(u)$ represents the present value of the accumulated capital gains at time t . If the trading strategy $\{\phi(t), \delta(t)\}$ were self-financing, then it follows from (2.6) that

$$d[CO(t)] = d[PV(t)] - \delta(t)dV(t) = 0.$$

In this case, $CO(t)$ would be a constant and $CO(t) = CO(0) = PV(0)$. Obviously, the trading strategy $\{\phi(t), \delta(t)\}$ under the present consideration is not self-financing due to the embedded mortality risk associated with the payoff $C(t)$. Hence, we seek a trading strategy $\{\phi(t), \delta(t)\}$ which minimizes the variance of the cost process $\{CO(t)\}$ under the risk-neutral probability measure. A strategy of this kind is called the risk-minimizing trading strategy (see for example Schweizer, 1994, or Møller, 1998). More precisely, the risk-minimizing trading strategy $\{\phi(t), \delta(t)\}$ is the optimal stochastic process for the following optimization problem:

$$\text{Minimize}_{\phi, \delta} E \left[\left(CO(t) - E[CO(t)] \right)^2 \right], \text{ for all } 0 \leq t \leq \mathcal{T}. \quad (3.3)$$

Schweizer (1994) (see also Møller, 1998) shows that the optimal solution of the optimization problem, $PV(t)$, $\delta(t)$ and $\phi(t)$ are given as follows:

- For $t = 0, 1, \dots, \mathcal{T}$,

$$\begin{aligned} PV(t) &= E \{C \mid \mathcal{F}_t\} = E \{E[C \mid T(x)] \mid \mathcal{F}_t\} \\ &= \sum_{j=0}^{t-1} e^{-\int_0^{j+1} r(u)du} C(j+1) I(j < T(x) \leq j+1) \\ &\quad + I(T(x) > t) e^{-\int_0^t r(u)du} \sum_{j=t}^{\mathcal{T}-1} \Pi(t, j+1) {}_{j-t}p_{x+t} q_{x+j} \\ &\quad + I(T(x) > t) e^{-\int_0^t r(u)du} \Pi(t, \mathcal{T}) {}_{\mathcal{T}-t}p_{x+t}, \end{aligned} \quad (3.4)$$

where $\Pi(t, j)$ as defined in (2.5). If time t is between two payment dates; i.e., there is an integer k such that $k < t < k+1$, then $PV(t)$ is obtained by (i) replacing t by k in the summation limits, (ii) adding $e^{-\int_0^t r(u)du} \Pi(t, k+1) I(k < T(x) \leq t)$, and (iii) replacing q_{x+t} by ${}_{k-t+1}q_{x+t}$ in the first term of the second summation of (3.4). For the special case where $t = 0$, the above expression simplifies to

$$PV(0) = \sum_{j=0}^{\mathcal{T}-1} \Pi(0, j+1) {}_j p_x q_{x+j} + \Pi(0, \mathcal{T}) {}_{\mathcal{T}} p_x. \quad (3.5)$$

- For $t = 0, 1, \dots, \mathcal{T}$,

$$\delta(t) = I(T(x) > t) \left[\sum_{j=t}^{\mathcal{T}-1} \delta(t, j+1) {}_{j-t}p_{x+t} q_{x+j} + \delta(t, \mathcal{T}) {}_{\mathcal{T}-t}p_{x+t} \right], \quad (3.6)$$

where $\delta(t, j)$ as given in (2.6). Similarly, if $k < t < k + 1$, the above equation becomes

$$\begin{aligned} \delta(t) = & I(T(x) > t) \left[\delta(t, k + 1) {}_{k-t+1}q_{x+t} \right. \\ & \left. + \sum_{j=k+1}^{\mathcal{T}-1} \delta(t, j + 1) {}_{j-t}p_{x+t} {}_q_{x+j} + \delta(t, \mathcal{T}) {}_{\mathcal{T}-t}p_{x+t} \right]. \end{aligned} \quad (3.7)$$

- Finally, $\phi(t) = e^{\int_0^t r(u)du} PV(t) - \delta(t)S(t)$.

To conclude this section, we remark that the above results can be generalized to benefits that are payable at the moment of death. In this case, expressions (3.4) and (3.6) respectively become

$$\begin{aligned} PV(t) = & \int_0^t e^{-\int_0^s r(u)du} C(s) dI(T(x) \leq s) \\ & + I(T(x) > t) e^{-\int_0^t r(u)du} \int_t^{\mathcal{T}} \Pi(t, s) {}_{s-t}p_{x+t} \mu(x + s) ds \\ & + I(T(x) > t) e^{-\int_0^t r(u)du} \Pi(t, \mathcal{T}) {}_{\mathcal{T}-t}p_{x+t}, \end{aligned}$$

and

$$\delta(t) = I(T(x) > t) \left[\int_t^{\mathcal{T}} \delta(t, s) {}_{s-t}p_{x+t} \mu(x + s) ds + \delta(t, \mathcal{T}) {}_{\mathcal{T}-t}p_{x+t} \right].$$

4 Pricing EIAs

In this section, we focus on pricing and reserving issues of EIAs using the framework presented in the previous section. As we have discussed earlier, an EIA is priced through its participation to the index growth. For an EIA with an initial value of one monetary unit, the payoff $C(t)$ at time t and hence $\Pi(0, t)$, are both functions of the EIA's participation rate or the spread between the index return and the EIA's credited return. For example, a vanilla point-to-point EIA has the following payoff structure:³

$$C(t) = \max \left[\min[1 + \alpha R_t, (1 + \zeta)^t], \beta(1 + g)^t \right], \quad (4.1)$$

where R_t measures the ‘‘appreciation’’ of the index up to year t , the participation rate α determines the proportion of the growth of the index fund to be credited, the cap or ceiling rate ζ is the annualized maximum rate that can be credited, the parameter g is the minimum annualized guarantee rate (or the floor rate) for the entire term of the contract, and β is the percentage of unit for which the minimum guarantee is applied to. Consequently, the present value of the EIA (i.e. $PV(0)$ as given in (3.5)) is also a function of the participation

³This and other EIAs will be discussed in details in the next section.

rate. As the initial value of the EIA is equal to 1, we have $PV(0) = 1$ and it follows from (3.5) that the participation rate α is a solution to the following pricing equation:

$$\sum_{j=0}^{\mathcal{T}-1} \Pi(0, j+1) {}_j p_x q_{x+j} + \Pi(0, \mathcal{T}) {}_{\mathcal{T}} p_x = 1 \quad (4.2)$$

We call the implied participation rate satisfying the above equation the critical or the fair participation rate. By holding all other parameter values constant, the critical participation rate always exists since the expression on the left-hand-side of (4.2) is an increasing function in α . Also the pricing formula (4.2) implies that the calculation is based on a net premium basis, meaning that the risk-minimizing hedging strategy discussed in the last section is employed or the mortality risk is diversified by selling enough policies of the same kind, and other charges such as the management and expenses charges are not taken into account. The same argument may be used to set reserves over time. Let $R(t)$, for $t = 0, 1, \dots, \mathcal{T}$, be the reserve at time t . Under the risk-minimizing hedging strategy, we have $R(t) = e^{\int_0^t r(u) du} E \{C \mid T(x) > t\}$ so that this quantity can be computed as

$$R(t) = \sum_{j=t}^{\mathcal{T}-1} \Pi(t, j+1) {}_{j-t} p_{x+t} q_{x+j} + \Pi(t, \mathcal{T}) {}_{\mathcal{T}-t} p_{x+t}. \quad (4.3)$$

An obvious drawback of the above approach is that the mortality risk is not well addressed, particularly when dealing with a small number of policies. To partly overcome this problem, we may consider the use of the percentile premium principle described in Bowers, et. al. (1997, Chapter 6). Consider a portfolio of n independent and identical policies. Under this principle, we have the following alternative pricing formula:

$$E(C) + (\varepsilon/\sqrt{n}) \sqrt{Var\{E[C \mid T(x)]\}} = 1, \quad (4.4)$$

where C is the representative of the present value of any of these policies' payoff, and ε is, say, the 95th percentile of the distribution of the normalized random variable of $E[C \mid T(x)]$. For a small value of n such as $n = 1$, the parameter ε may be estimated using the smoothed empirical estimation of ε (see Klugman, Panjer and Willmot, 1998, p. 35). For a relative large n (say $n \geq 20$) a normal approximation of the distribution would be appropriate and we may let $\varepsilon = 1.96$. We refer the participation rate implied from (4.4) as the loaded participation rate. Intuitively, the loaded participation rate guarantees that for each policy, there is sufficient fund covering EIA benefits 95% of the time. Consequently, a loaded participation rate from (4.4) is always lower than the critical participation rate from (4.2). However as n increases, the loaded participation rate approaches the critical participation rate.

The calculation of loaded participation rate does not require much additional effort relative to computing the critical participation rate that satisfies (4.2). This follows since

$$Var\{E[C \mid T(x)]\} = E\{[E[C \mid T(x)]]^2\} - [E(C)]^2$$

and

$$E \{ [E [C | T(x)]]^2 \} = \sum_{j=0}^{\mathcal{T}-1} [\Pi(0, j+1)]^2 {}_j p_x q_{x+j} + [\Pi(0, \mathcal{T})]^2 \tau p_x,$$

and the quantities $\Pi(0, s)$ and hence $E(C) = PV(0)$ are already known from computing the critical participation rate. The corresponding loaded reserve is then obtained via

$$R_L(t) = e^{\int_0^t r(u) du} \left\{ E(C | T(x) > t) + (\varepsilon/\sqrt{n}) \sqrt{\text{Var}\{E [C | T(x+t)]\}} \right\}. \quad (4.5)$$

Both (4.2) and (4.4) are the basis for the numerical implementation in the next section. We note that the risk-neutral probability measure is used in the above formulas as well as in the numerical implementation when calculating the conditional expectations, and the mortality law is used to calculate the outside expectations (see Footnote 2 for an explanation).

5 Numerical Implementation

In this section, we provide numerical analysis on the model. We achieve this by considering several types of EIAs. Our examples involve 7-year EIAs issuing to a life aged 58 with minimum interest rate guarantee of either 3% on 100% of the premium or 3% on 90% of the premium. In other words, we assume that the annuitant aged 58 would hold the policy for 7 years, contingent on survival, and convert it into a traditional fixed annuity at retirement age 65. The mortality of the annuitant is assumed to follow the 1979-1981 U.S. Life Table (see Table 3.3.1 of Bowers et al. (1997)). The index fund in consideration is governed by a geometric Brownian motion with the initial value normalized to 1 unit. The volatility of the index is assumed to be constant and is either 20% or 30%. For the interest rate model (2.1), we use the same parameter values estimated by Ait-Sahalia (1996), i.e., $\kappa = 0.85837$ and $\theta = 0.089102$. This implies that the long term mean interest rate is around 8.9% and the mean-reverting intensity is 0.85837. We also assume the initial interest rate $r_0 = 0.05$ and the volatility of the interest rate takes the values of {0%, 4%, 8%} to reflect the current interest rate environment and to examine the impact of the volatility of the interest rate. Note that although zero volatility represents that the interest rate is deterministic, the rates need not be constant over time. Hence our numerical results also provide a comparison between the valuation under deterministic interest rate and the valuation under stochastic interest rate. In the following subsections, we will first consider the critical participation rate (or the critical spread or cap as will be seen later) based on pricing formula (4.2) and the loaded participation rate under the alternative pricing formula (4.4).

5.1 Point-to-Point EIAs

In this subsection, we consider one of the simplest classes of EIAs. The crediting strategy for this family is known as the *Point-to-Point* design and in general, its contingent claim

$C(t)$ in year t for one unit of EIA can be represented as that in (4.1); i.e.,

$$C(t) = \max \left[\min[1 + \alpha R_t, (1 + \zeta)^t], \beta(1 + g)^t \right]. \quad (5.1)$$

The above payoff structure is ideal for many investors. While subject to the maximum cap rate that can be earned under this design, the first random term in (5.1) allows the investors to participate in any potential upside gain in the equity market. More importantly, in the event of an adverse market environment, the downside risk is constrained to the minimum guarantee floor component; i.e. $\beta(1 + g)^t$. The presence of the cap rate, although placing an upper bound on the rate of return of the contract, could reduce the cost of such design substantially. In this subsection, we will first focus on identifying a critical participation rate, while fixing g, β and ζ . There exists various variations of Point-to-Point EIAs depending on how we define the equity return random variable R_t in (5.1).

We now discuss several structures of R_t . As R_t denotes the “gain” for the reference index over the time interval $(0, t]$, without loss of generality, we can redefine R_t as

$$R_t = \frac{S^*(t)}{S(0)} - 1, \quad (5.2)$$

where $S^*(t)$ is an appropriate variable yet to be determined. Using this representation, the variable $C(t)$ can be formulated as the following two equivalent expressions:

$$C(t) = \beta(1 + g)^t + \frac{\alpha}{S(0)} \left\{ \max[S^*(t) - K_g, 0] - \max[S^*(t) - K_\zeta, 0] \right\} \quad (5.3)$$

or

$$C(t) = (1 + \zeta)^t - \frac{\alpha}{S(0)} \left\{ \max[K_\zeta - S^*(t), 0] - \max[K_g - S^*(t), 0] \right\} \quad (5.4)$$

where

$$K_\zeta = \frac{S(0)}{\alpha} [(1 + \zeta)^t - 1 + \alpha]$$

and

$$K_g = \frac{S(0)}{\alpha} [\beta(1 + g)^t - 1 + \alpha].$$

These expressions provide additional insight on hedging the EIA products. The first representation (5.3) suggests that one hedging approach is to construct a portfolio with the following three positions: (i) a zero-coupon bond which matures to $\beta(1 + g)^t$ in year t , (ii) a long position of $\frac{\alpha}{S(0)}$ units of call options with strike price K_g , and (iii) a short position of $\frac{\alpha}{S(0)}$ units of call options with strike price K_ζ . The hedging strategy involving (ii) and (iii) is often referred to as a bull spread, a common practice in option trading (see Hull, 2000, pp. 187-189). Alternatively from (5.4), an equivalent strategy is to simultaneously invest in

a t -year zero-coupon maturing to $(1 + \zeta)^t$ and a short bear spread (i.e., a long and a short position of $\frac{\alpha}{S(0)}$ units of put options with strike prices K_g and K_ζ).

Note that the above formulations can easily be adjusted for a design that does not place an upper bound on the rate of return. This is accomplished by setting $\zeta = \infty$ so that $\max[S^*(t) - K_\zeta, 0] = 0$ and (5.3) simplifies to

$$C(t) = \beta(1 + g)^t + \frac{\alpha}{S(0)} \max[S^*(t) - K_g, 0]. \quad (5.5)$$

Similarly we have $\max[K_\zeta - S^*(t), 0] = K_\zeta - S^*(t)$ when $\zeta = \infty$ so that (5.4) reduces to

$$C(t) = 1 - \alpha + \frac{\alpha}{S(0)} \left\{ S^*(t) + \max[K_g - S^*(t), 0] \right\}. \quad (5.6)$$

The above analysis implies that the pricing of the EIAs with Point-to-Point design boils down to the valuation of the corresponding call (or put) options. This in turn depends on how the key variable $S^*(t)$ is determined. In practice, various forms of $S^*(t)$ have been proposed. In this paper, we consider three of the most popular structures, namely the long-term or term-end *Point-to-Point* design, the *Asian-End* design, and the *High-Water-Mark* or *Term-High Point* design. We now discuss these features in greater details.

The Term-End Point-to-Point design, which is also the simplest crediting method, defines $S^*(t)$ as the observed index value at the end of the year of surrendering the contract; i.e.,

$$S^*(t) = S(t). \quad (5.7)$$

This particular case is easy to analyze since the cost of the EIA reduces to a portfolio of standard call (or put) options. Analytic solutions of these options are possible under certain interest rate models including the Vasicek model.⁴ The results in Tables 1, 2 and 3 are obtained analytically.

Table 1 compares the critical and loaded participation rates implied from (4.2) and (4.4), respectively, over various sets of parameter values. For the loaded case, we consider $n = 20$ and 100. In all these cases, the EIAs are based on 7-year Term-End point crediting designs without the ceiling cap rate. We also consider two types of minimum guarantee: one with $g = 3\%$ and $\beta = 100\%$ and the other with $g = 3\%$ and $\beta = 90\%$. Based on these results, we make the following remarks:

- 1a. As expected, the loaded participation rates are consistently lower than the corresponding unloaded critical participation rates.
- 1b. The critical (or loaded) participation rates for cases with $\beta = 100\%$ are consistently lower than the corresponding values on 90% premium. With the lower proportion guarantee on the premium, the cost of EIAs must be lower. Thus the participation rates must increase to raise the cost of the EIAs to unity.

⁴The pricing formulas for equity European call or put options with Vasicek interest rate process are given in Rabinovitch (1989).

Table 1: Point-to-point with Term-End design (without cap): Impact of correlation, volatilities of index and interest rates on the critical and loaded participation rates

σ_S	σ_r	Critical Participation Rates using (4.2)			Loaded Participation Rates using (4.4)					
		Correlation			with $n = 20$			with $n = 100$		
		-30%	0	30%	-30%	0	30%	-30%	0	30%
3% Min. Guarantee on 100% Premium										
20%	0%	85.95	85.95	85.95	84.88	84.88	84.88	85.49	85.49	85.49
	4%	86.62	85.04	83.55	85.57	83.98	82.49	86.17	84.58	83.09
	8%	85.42	82.25	79.45	84.39	81.21	78.42	84.98	81.80	79.00
30%	0%	73.32	73.32	73.32	72.03	72.03	72.03	72.76	72.76	72.76
	4%	73.89	72.54	71.28	72.60	71.27	70.02	73.33	71.99	70.73
	8%	72.86	70.16	67.79	71.60	68.94	66.60	72.31	69.63	67.28
3% Min. Guarantee on 90% Premium										
20%	0%	90.89	90.89	90.89	90.40	90.40	90.40	90.68	90.68	90.68
	4%	91.49	90.20	88.97	91.00	89.71	88.47	91.28	89.99	88.75
	8%	90.67	88.05	85.66	90.21	87.57	85.18	90.47	87.84	85.45
30%	0%	79.94	79.94	79.94	79.21	79.21	79.21	79.62	79.62	79.62
	4%	80.52	79.29	78.14	79.79	78.57	77.43	80.20	78.98	77.83
	8%	79.80	77.32	75.12	79.10	76.64	74.46	79.50	77.03	74.83

- 1c. As we increase the volatility of the index fund, the critical (or loaded) participation rate declines. This is to be expected due to the fact that the more volatile the fund is, the greater the appreciation of the fund. Since the value of the EIA must remain at unity, the critical (or loaded) participation rate must necessarily be lower in order to compensate the greater gain from the index. An alternate argument is to revisit the option decompositions in (5.5) or (5.6). For instance, let us consider (5.5) which involves the embedded call option. The value of a call option increases with the volatility of the underlying index. Hence to dampen the increased value of the call option, the strike price K_g is adjusted upward by reducing the participation rate α in order that to satisfy (4.4) or (4.2). Also note that the participation rate is sensitive to the index volatility.
- 1d. Holding all other parameters constant, the critical (or loaded) participation rate is a non-increasing function in ρ . This follows from the property of the call option that its value is non-decreasing function in ρ .
- 1e. The results corresponding to $\sigma_r = 0$ represent an economy with deterministic interest rates. This is a generalization of Black-Scholes framework in that the implied yield curve from Vasicek model with $\sigma_r = 0$ is not necessary flat. In fact for the parameter values considered in our examples, the term structure of interest rates is upward sloping. These values, therefore, provide a benchmark when we induce randomness to the underlying interest rates.

1f. For $\rho \geq 0$, the critical (or loaded) participation rate declines monotonically as the volatility of the interest rate increases. On the other hand, for the negatively correlated cases, the critical (or loaded) participation rates increase initially with the volatility of the interest rates. As the volatility of the interest rate increases further, the critical (or loaded) participation rate drops. This phenomenon is attributed to the following two non-linear relationships: (i) first for the sets of parameter values considered in these examples, as σ_r increases, the value of the call option in (5.5) increases for $\rho \geq 0$ while decreases for $\rho < 0$. (ii) second, although the implied yield curves are upward sloping regardless of the volatility of the interest rates, the corresponding spot rates decline as the interest rates become more volatile. These two properties lead to the empirical behavior of the critical participation rates. For $\rho < 0$, there are two contradicting effects simultaneously influencing the value of an EIA. Property (i) leads to a decline in the value of the option as interest rate volatility increases. At the same time, property (ii) implies that the spot rates will drop, inflating the expected value component involving the minimum guarantee term in (5.5). The net impact is that with an initial decrease in the volatility of the interest rate, the first effect dominates and hence raises the participation rate to ensure that relation (4.2) (or (4.4)) remains valid. However, as the interest rate continues to be more volatile, the second effect becomes more pronounced and consequently leads to a decrease in the fair participation rate.

Table 2: Point-to-point with Term-End design and 20% cap rate: Impact of correlation, volatilities of index and interest rates on the critical and loaded participation rates

σ_S	σ_r	Critical Participation Rates using (4.2)			Loaded Participation Rates using (4.4)					
		Correlation			with $n = 20$			with $n = 100$		
		-30%	0	30%	-30%	0	30%	-30%	0	30%
3% Min. Guarantee on 100% Premium										
20%	0%	91.22	91.22	91.22	90.42	90.42	90.42	90.88	90.88	90.88
	4%	91.03	90.48	90.06	90.25	89.68	89.23	90.70	90.13	89.71
	8%	89.38	88.10	87.32	88.61	87.28	86.44	89.05	87.75	86.94
30%	0%	91.66	91.66	91.66	90.44	90.44	90.44	91.14	91.14	91.14
	4%	90.17	90.69	91.33	89.01	89.47	90.04	89.67	90.16	90.77
	8%	86.98	87.63	88.73	85.87	86.42	87.40	86.50	87.11	88.16
3% Min. Guarantee on 90% Premium										
20%	0%	97.48	97.48	97.48	97.33	97.33	97.33	97.41	97.41	97.41
	4%	97.00	97.07	97.25	96.86	96.92	97.10	96.93	97.00	97.18
	8%	95.73	95.65	96.01	95.59	95.50	95.84	95.66	95.58	95.93
30%	0%	104.36	104.36	104.36	104.07	104.07	104.07	104.23	104.23	104.23
	4%	102.21	103.64	105.21	101.93	103.35	104.90	102.08	103.50	105.07
	8%	98.91	101.31	104.24	98.65	101.01	103.91	98.79	101.17	104.09

We now consider the impact on the critical and loaded participation rates by exerting

a ceiling on the rates that can be credited. Comparing to an otherwise identical design, the presence of a cap should results in an increase in the critical and loaded participation rates. Furthermore, the magnitude of the increment should be more pronounced for more volatile market environment. These intuitions are confirmed by the results shown in Table 2 which are based on the same set of examples as in Table 1 except by enforcing a 20% cap rate. As an illustration, consider a low-volatility case with $\sigma_S = 20\%$, $\sigma_r = 4\%$ and a high-volatility case with $\sigma_S = 30\%$, $\sigma_r = 8\%$, the critical participation rates for $\rho = 30\%$ and 3% minimum guarantee on 90% of the premium are 88.97% and 75.12%, respectively. By confining the crediting rate to a 20% cap rate, the critical participation rates for the low- and high-volatility cases are 97.25% and 104.24%, representing increment of approximately 9% and 39%, respectively. Hence in both cases the critical participation rates increase but the magnitude of the compensation is significantly greater for the more volatile situation.

Besides the increase in the critical (or loaded) participation rates, there are other notable non-linear impact on the implied participation rates. These effects can be explained by comparing (5.3) to (5.5) (or equivalently comparing (5.4) to (5.6)). For instance for the design without a cap, we observed that as the correlation increases, the critical (or loaded) participation rate is non-increasing (see remark 1d). This is attributed to the non-decreasing function of the underlying call option as we increase the correlation. The presence of cap complicates this behavior considerably. Observe that from Table 2 as we increase the correlation, the critical (or loaded) participation rates can remain unchanged, decrease or even increase. The non-linearity arises because with an upper bound on the crediting rate, its value depends on the difference of two call options (see (5.3)) and the difference of two non-decreasing functions is not necessary non-decreasing.

Table 3: Impact on critical and loaded participation rates by varying cap rates in a Term-End Point-to-Point design with $\sigma_S = 20\%$, $\sigma_r = 4\%$, $g = 3\%$ and $\beta = 100\%$

Cap Rates (%) ζ	Critical Participation Rates using (4.2)			Loaded Participation Rates using (4.4)					
				with $n = 20$			with $n = 100$		
	Correlation			Correlation			Correlation		
	-30%	0	30%	-30%	0	30%	-30%	0	30%
∞	86.62	85.04	83.55	85.57	83.98	82.49	86.17	84.58	83.09
50	86.63	85.05	83.56	85.58	84.00	82.51	86.18	84.59	83.11
45	86.64	85.05	83.57	85.60	84.02	82.53	86.19	84.60	83.12
40	86.65	85.07	83.60	85.63	84.05	82.58	86.21	84.63	83.16
35	86.69	85.14	83.70	85.70	84.15	82.71	86.27	84.71	83.27
30	86.89	85.42	84.07	85.95	84.48	83.13	86.48	85.01	83.66
25	87.70	86.48	85.39	86.83	85.60	84.51	87.32	86.10	85.01
20	91.03	90.48	90.06	90.25	89.68	89.23	90.70	90.13	89.71
18	94.46	94.47	94.61	93.72	93.69	93.78	94.15	94.14	94.25
16	101.02	102.01	103.12	100.30	101.22	102.27	100.71	101.67	102.75
14	115.09	118.19	121.52	114.36	117.36	120.56	114.78	117.84	121.11
12	155.34	166.09	178.05	154.48	164.99	176.66	154.97	165.62	177.45

Table 3 considers the effect on the implied participation rates as we change the cap rates. These examples are based on $\sigma_S = 20\%$, $\sigma_r = 4\%$ and $\rho = -30\%, 0, 30\%$. First note that the results corresponding to $\zeta = \infty$ represent the design without the cap. Second, as we decrease the cap rates, the critical participation rates increase. The participation rates increase gradually initially but accelerate at an increasing speed as the cap rates reduce further. For instance consider the positively correlated case, the critical participation rate increased by 5% when the cap rate drops from 20% to 18%. On the other hand, when the cap rate reduces further from 14% to 12%, the increment is approximately 47%. Third, it was pointed out that as we raise the correlation, the participation rates can behave differently depending on the set of parameter values. This is similarly observed as we modify the cap rates.

The design in Term-End Point can be criticized on the ground that the rate of return process depends solely on the index level at the end of the term. In the event of a sudden adverse movement in the equity market, this feature can be devastating to investors. To mitigate the risk of depending the index level on one particular day, an averaging scheme has been proposed. This design is commonly referred as the *Asian-End* since it depends on the daily, weekly or monthly index levels in the year when the claim is surrendered. For instance for an Asian-End indexing method with monthly averaging, the variable S^* in (5.2) is defined as:

$$S^*(t) = \frac{\sum_{k=0}^{11} S(t - \frac{k}{12})}{12}. \quad (5.8)$$

Since the averaging design is less volatile than the design which depends only on a single observation, this implies that the value of an EIA with Asian-End is cheaper than the corresponding EIA with Term-End Point.

Another more exotic structure which has appeared in EIAs is the *Term-High* or *High-Water-Mark* design. In this case, the variable S^* corresponds to the highest realized index level over the term of the contract. The sampling frequency of the index levels is typically daily, monthly or annually. Naturally, the higher the frequency of sampling, the more expensive the EIAs. Also, the High-Water-Mark design is more expensive than the corresponding EIAs with Term-End Point design, which in turn is more costly than the EIA with Asian-End.

It should be emphasized that although the variables S^* in both Asian-End and High-Water-Mark designs are defined differently than that in Term-End Point design, expressions (5.3) and (5.4) for EIA without cap and expressions (5.5) and (5.6) for EIA with cap are still applicable. Hence the values of EIAs reduce to computing Asian options and lookback options for the Asian-End and High-Water-Mark designs, respectively. It is well-known that a simple closed-form expression does not exist for the Asian options.⁵ For the lookback options with discrete sampling points, their values can be expressed in terms of the multi-

⁵A closed-form solution for the Asian option exists but it is very complicated. It involves inversion of a nontrivial Laplace transform and is practically not feasible. See Geman and Yor (1993).

variate normal probabilities under the Black-Scholes framework with constant interest rate assumption. Efficient approaches for handling this problem are discussed in Lin (1999) and Boyle, Lai and Tan (2001). For our model with the additional complexity that the interest rates are stochastic, we resort to simulation to compute the value of the EIAs.

Table 4: Point-to-point with Asian-End design (on last 12 months' index): Impact of correlation, volatilities of index and interest rates on the critical and loaded participation rates

Critical Participation Rates based on (4.2)				
σ_S	σ_r	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium				
20%	0%	94.27(0.11)	94.27(0.11)	94.27(0.11)
	4%	94.89(0.11)	93.28(0.11)	91.76(0.10)
	8%	93.49(0.12)	90.25(0.11)	87.39(0.10)
30%	0%	80.16(0.15)	80.16(0.15)	80.16(0.15)
	4%	80.64(0.15)	79.32(0.15)	78.08(0.14)
	8%	79.39(0.16)	76.75(0.15)	74.43(0.13)
3% Minimum Guarantee on 90% Premium				
20%	0%	99.73(0.11)	99.73(0.11)	99.73(0.11)
	4%	100.25(0.12)	98.98(0.11)	97.75(0.11)
	8%	99.26(0.12)	96.64(0.11)	94.25(0.10)
30%	0%	87.35(0.16)	87.35(0.16)	87.35(0.16)
	4%	87.82(0.17)	86.65(0.16)	85.55(0.15)
	8%	86.89(0.17)	84.52(0.16)	82.43(0.15)

Loaded Participation Rates using (4.4)							
σ_S	σ_r	with $n = 20$			with $n = 100$		
		$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium							
20%	0%	93.67(0.10)	93.67(0.10)	93.67(0.10)	94.01(0.10)	94.01(0.10)	94.01(0.10)
	4%	94.31(0.11)	92.69(0.10)	91.16(0.10)	94.64(0.11)	93.02(0.10)	91.50(0.10)
	8%	92.93(0.11)	89.67(0.10)	86.81(0.09)	93.25(0.12)	90.00(0.11)	87.14(0.09)
30%	0%	79.27(0.14)	79.27(0.14)	79.27(0.14)	79.77(0.14)	79.77(0.14)	79.77(0.14)
	4%	79.75(0.15)	78.45(0.14)	77.23(0.13)	80.25(0.15)	78.94(0.14)	77.71(0.14)
	8%	78.53(0.15)	75.92(0.14)	73.63(0.13)	79.02(0.16)	76.39(0.14)	74.08(0.13)
3% Minimum Guarantee on 90% Premium							
20%	0%	99.51(0.11)	99.51(0.11)	99.51(0.11)	99.63(0.11)	99.63(0.11)	99.63(0.11)
	4%	100.03(0.12)	98.75(0.11)	97.51(0.11)	100.15(0.12)	98.87(0.11)	97.64(0.11)
	8%	99.03(0.12)	96.39(0.11)	94.00(0.10)	99.15(0.12)	96.53(0.11)	94.13(0.10)
30%	0%	87.00(0.15)	87.00(0.15)	87.00(0.15)	87.19(0.16)	87.19(0.16)	87.19(0.16)
	4%	87.48(0.16)	86.31(0.15)	85.21(0.15)	87.67(0.16)	86.50(0.16)	85.40(0.15)
	8%	86.56(0.17)	84.20(0.15)	82.11(0.14)	86.74(0.17)	84.38(0.16)	82.29(0.14)

In our numerical illustration, we also consider the same sets of parameter values as in Table 1. For each set of parameter values, we simulate 100,000 trajectories where each trajectory corresponds to the joint processes $\{S(t)\}$ and $\{r(t)\}$ being simulated daily (assuming 252 trading days per year). The critical and participation rates are then estimated from these trajectories using the bisection approach until (4.2) and (4.4) are satisfied. The above procedure is replicated independently 10 times to provide an estimate of the standard

errors for the estimated implied participation rates. Tables 4 and 5 provide comparison on the critical and loaded participation rates for EIAs without cap and with Asian-End and High-Water-Mark designs, respectively. The values in the brackets are the corresponding estimated standard errors. The sampling frequency for both designs are monthly so that the S^* in Asian-End is defined according to (5.8) while in High-Water-Mark we have

$$S^*(t) = \max_{1 \leq k \leq 12t} S\left(\frac{k}{12}\right). \quad (5.9)$$

Observe that the critical (or loaded) participation rates for EIAs with Asian-End design are consistently higher than the respective rates in Term-End Point design, which in turn are higher than those from High-Water-Mark designs. This is consistent with the argument made earlier that among the three designs, Asian-End is the least expensive while the High-Water-Mark is the most expensive. Note that the degree of penalty in the critical (or loaded) participation for the High-Water-Mark design is large relative to both Asian-End and Term-End Point designs, particularly for more volatile cases. Apart from this, the behavior of the implied participation rates in these designs is very similar to those with Term-End point design.

5.2 Annual Reset/Ratchet EIAs

In this subsection, we focus on the most popular type of EIAs known as the Annual Reset or Annual Ratchet EIAs. According to Marrion (2000, 2001), these products comprise of nearly 70% of EIAs sold in the marketplace.⁶ Generically, its payoff in year t for one unit of EIA is given by

$$C(t) = \max \left\{ \prod_{s=1}^t \max \left[\min[1 + \alpha R_s - \gamma, 1 + \zeta], 1 \right], \beta(1 + g)^t \right\}, \quad (5.10)$$

where the random variable R_s again measures the appreciation of the referenced index fund in year s , the yield spread γ is the flat rate that is deducted from the gain in the index fund each year. The remaining parameters α, ζ, β , and g are defined similarly as in (5.1).

Variations of Annual Reset EIAs exist depending on the precise definition of the random variable R_s . The most common type of design is based on averaging, which according to Marrion (2000, 2001) represent approximately 60% of Annual Reset EIAs. One possible averaging approach is based on the monthly index levels with $R(s)$ defined as

$$R_s = \frac{1}{S(s-1)} \frac{\sum_{k=0}^{11} S(s - \frac{k}{12})}{12} - 1. \quad (5.11)$$

⁶Here we consider all EIAs with annual resetting feature, including annual yield spread EIAs that will be discussed at the end of this subsection.

Another popular design is the Term-End point. In this case, the random variable R_s is solely determined by the index levels at the beginning and at the end of year s through the following structure:

$$R_s = \frac{S(s)}{S(s-1)} - 1. \quad (5.12)$$

Comparing Annual Reset EIAs to Point-to-Point EIAs, the former can be more appealing to the investors for two reasons. First is that the interest is credited each year for the Annual Reset EIAs. The credited interest cannot be lost even if the index subsequently goes down. Second, the index level used to determine the appreciation of the index is reset annually. This “lock in” feature can be extremely valuable, particularly in a more volatile market. For instance, let us consider an Annual Reset EIA with Term-End Point design and also assume that the index drops by 20% in the first year. By the nature of the design, no interest is credited in the first year but the index appreciation for the second year would be based on the lower level. This implies that even if the index returns to its initial level in the following year, the crediting rate would become 25% for that year. For these reasons, Annual Reset EIAs would be more expensive than the corresponding EIAs with Point-to-Point designs. Note that the values of the Annual Reset EIAs increase with the parameters β, g, α, ζ while decrease with the yield spread γ . This provides ways for reducing the cost of these annuities.

Similar to the Point-to-Point EIAs, we assess the cost of the Annual Reset EIAs by computing the implied participation rates from both (4.2) and (4.4) for a given set of parameters β, g, ζ, γ . Due to the complexity of the designs, we use simulation to estimate the implied rates. We consider three typical types of EIAs. The first type is the Annual Reset EIAs with Term-End Point design and with $\zeta = \infty, \gamma = 0$; i.e. no cap and no yield spread. The second type is similar to the first except with a ceiling of 20%. For the third case, we consider Annual Reset EIAs without cap and yield spread but with monthly averaging indexing method; i.e. the index gain is defined according to (5.11). The results are reported in Tables 6, 7 and 8. The assumptions for the underlying index and interest rate processes are identical to the last subsection. The implied participation rates are also computed based on 10 independent replications with 100,000 trajectories in each sample. The behavior of the implied participation rates by changing the volatilities of the index, interest rate, the correlation and across different types of designs are very similar to those observed in Point-to-Point EIAs. We summarize the notable differences as follows:

- 2a. The ratchet feature for the EIAs with Term-End Point design can be quite expensive. For example, for the Term-End Point-to-Point design (without cap) with minimum guarantee applied to 100% of the premium, the critical participation rates are around 85% and 72% for $\sigma_s = 20$ and 30%, respectively. On the other hand, by changing the crediting strategy to a ratchet design, the fair participation rates for the respective cases need to be reduced substantially to around 63% and 49%.

- 2b. Unlike the Point-to-Point EIAs, the critical participation rates for the minimum guarantee on 90% of the premium are only slightly larger than the corresponding cases guaranteeing 100% of the premium. This suggests that whether to have the minimum guarantee on 90% or on full premium, the associated cost is insignificant.
- 2c. The impact for enforcing a ceiling on the crediting rate is of greater magnitude for the EIAs with reset feature. In some cases, the increase in the implied participation rates are almost twice. This indicates that putting a ceiling on the credited rate significantly reduces the value of EIA and hence the policyholder needs to be compensated through a substantial increase in the participation rates.
- 2d. By switching the indexing method from Term-End point to averaging, the critical (or loaded) participation rates should increase and this is consistent with our results (compare Table 1 to Table 4 and Table 6 to Table 8). The increment, however, is more significant for the Annual Reset EIAs.

In our analysis so far, we evaluate the costs of EIAs by comparing the implied participation rates. We now consider other ways of assessing the cost of EIAs. In theory, as long as all but one parameter are given, the remaining parameter value can always be implied from (4.2) (or (4.4)). For the examples we have considered, we chose to deduce the fair participation rates. Alternatively, we could let the cap rate be the “free” parameter and fix the participation rate as well as the remaining parameters in (4.2) (or (4.4)). The implied (critical or loaded) cap rate always exists and is also unique since the value of the EIA increases monotonically with the cap. Table 9 produces the critical cap rates for Annual Reset EIAs with Term-End Point indexing method. For brevity the loaded cap rates are not reported. These examples assume full participation and absence of yield spread; i.e. $\alpha = 1$ and $\gamma = 0$. The critical cap rates are consistent with the critical participation rates reported in Table 6. An immediate conclusion which can be drawn from this table is that the critical cap rate is insensitive to volatilities and correlation.

In all the examples above, we have assumed $\gamma = 0$ so that nothing is deducted from the gain. We now consider the case with nonzero yield spread. In fact, EIAs with nonzero yield spread are commonly known as the *Annual Yield Spread* EIAs. Furthermore, the participation rates in these cases are typically 100%. EIAs of this design represent a market share of around 33% and is the second most popular EIAs currently being offered. As in the critical participation rate and the critical cap rate, we can similarly assess the cost of the Annual Yield Spread EIAs by comparing the implied yield spread which satisfies (4.2). Note that in this case as we increase the yield spread, the value of the EIA reduces. Tables 10 and 11 are based on the same indexing strategies as Tables 6 and 8, respectively. In these cases, the critical yield spreads are computed by letting participation rate $\alpha = 100\%$ and cap rate $\zeta = \infty$.

Based on these results, we make the following observations:

- 3a. The critical yield spread is very sensitive to the volatility of the index. By increasing σ_S from 20% to 30%, the critical yield spread need to be compensated more than double.
- 3b. Averaging the index not only dampens its volatility, but also significantly reduces the critical yield spread. For instance, consider 30% volatility of the index, the critical yield spread drops from about 20% to 3% by switching the design from Term-End to averaging.
- 3c. Even with a volatility of 20% in the equity market, the critical yield spread for EIAs with averaging indexing method needs to be negative in order to satisfy (4.2). This implies that an EIA providing 3% minimum interest guarantee and 100% participation in the index can be offered at no additional cost with averaging design.

6 Concluding Remarks

In this paper, we introduce an economic model which not only captures the behavior of the equity index, but also the interest rates. This is an improvement over the traditional model which only permits the index to be stochastic while having the interest rates constant. A pricing formula for EIAs is obtained using the risk-minimization hedging strategy of Schweizer (1994). The pricing formula assumes that the mortality risk can be diversified and hence it might not be proper for companies that sell a small number of EIA policies. We thus extend the pricing formula using the percentile premium principle. The new pricing formula then allows to add a margin for the mortality risk. Reserving with both formulas are briefly discussed.

A detailed numerical analysis is then performed for valuing various existing EIAs in the North American market. The results from this analysis may be used as guidelines and benchmarks for insurance companies for the valuation of their EIAs. Furthermore, our methodology may be used to evaluate variable annuities (VAs), due to the similarity in payoff structure between EIAs and VAs. As a caution, it should be noted that our conclusion may only be relevant to the types of EIA we considered in this paper, particularly for the ranges of parameter values assumed. There are still some remaining issues related to the valuation of EIAs, which will be considered in our subsequent work. One important issue would be the identification of hedging strategies for these EIAs, involving bonds with various maturities and index options corresponding to the embedded guarantees.

We wish to point out that the early surrender/withdrawal risk is not discussed in this paper although it is important when pricing EIAs. The main difficulty of modeling the surrender/withdrawal risk is the lack of reliable data in this area. A common industrial practice is to assume a fixed percentage each year for the early surrender/withdrawal. In this case, our algorithms can easily be modified. However, it does not provide any new insight for the model and therefore is omitted.

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Table 5: Point-to-point with High-Water-Mark design (monthly monitoring): Impact of correlation, volatilities of index and interest rates on the critical and loaded participation rates

Critical Participation Rates based on (4.2)							
σ_S	σ_r	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$			
3% Minimum Guarantee on 100% Premium							
20%	0%	68.71(0.06)	68.71(0.06)	68.71(0.06)			
	4%	69.06(0.06)	67.89(0.06)	66.77(0.05)			
	8%	67.71(0.07)	65.38(0.06)	63.30(0.05)			
30%	0%	51.65(0.07)	51.65(0.07)	51.65(0.07)			
	4%	51.83(0.07)	51.05(0.07)	50.32(0.06)			
	8%	50.79(0.08)	49.26(0.07)	47.88(0.06)			
3% Minimum Guarantee on 90% Premium							
20%	0%	70.60(0.06)	70.60(0.06)	70.60(0.06)			
	4%	70.87(0.06)	69.87(0.06)	68.91(0.06)			
	8%	69.63(0.07)	67.66(0.06)	65.86(0.05)			
30%	0%	53.83(0.07)	53.83(0.07)	53.83(0.07)			
	4%	53.97(0.07)	53.29(0.07)	52.65(0.07)			
	8%	53.00(0.08)	51.67(0.07)	50.47(0.06)			
Loaded Participation Rates using (4.4)							
σ_S	σ_r	with $n = 20$			with $n = 100$		
		$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium							
20%	0%	67.72(0.05)	67.72(0.05)	67.72(0.05)	68.28(0.06)	68.28(0.06)	68.28(0.06)
	4%	68.07(0.06)	66.91(0.06)	65.82(0.05)	68.63(0.06)	67.46(0.06)	66.36(0.05)
	8%	66.74(0.07)	64.46(0.06)	62.42(0.05)	67.29(0.07)	64.98(0.06)	62.92(0.05)
30%	0%	50.75(0.06)	50.75(0.06)	50.75(0.06)	51.26(0.06)	51.26(0.06)	51.26(0.06)
	4%	50.92(0.07)	50.17(0.06)	49.46(0.06)	51.43(0.07)	50.67(0.07)	49.95(0.06)
	8%	49.91(0.07)	48.43(0.07)	47.10(0.06)	50.41(0.08)	48.90(0.07)	47.54(0.06)
3% Minimum Guarantee on 90% Premium							
20%	0%	69.73(0.06)	69.73(0.06)	69.73(0.06)	70.22(0.06)	70.22(0.06)	70.22(0.06)
	4%	69.99(0.06)	69.01(0.06)	68.08(0.05)	70.49(0.06)	69.50(0.06)	68.55(0.05)
	8%	68.78(0.07)	66.85(0.06)	65.09(0.05)	69.26(0.07)	67.31(0.06)	65.53(0.05)
30%	0%	53.02(0.06)	53.02(0.06)	53.02(0.06)	53.48(0.07)	53.48(0.07)	53.48(0.07)
	4%	53.15(0.07)	52.50(0.07)	51.89(0.06)	53.61(0.07)	52.95(0.07)	52.32(0.06)
	8%	52.21(0.07)	50.93(0.07)	49.77(0.06)	52.65(0.08)	51.35(0.07)	50.17(0.06)

Table 6: Annual Reset with Term-End point design (without cap): Impact of correlation, volatilities of index and interest rates on the critical and loaded participation rates

Critical Participation Rates based on (4.2)							
σ_S	σ_r	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$			
3% Minimum Guarantee on 100% Premium							
20%	0%	64.12(0.04)	64.12(0.04)	64.12(0.04)			
	4%	64.17(0.04)	63.53(0.04)	62.91(0.03)			
	8%	62.98(0.05)	61.73(0.04)	60.58(0.03)			
30%	0%	49.36(0.03)	49.36(0.03)	49.36(0.03)			
	4%	49.23(0.04)	48.94(0.03)	48.66(0.03)			
	8%	48.23(0.04)	47.66(0.04)	47.12(0.03)			
3% Minimum Guarantee on 90% Premium							
20%	0%	64.54(0.04)	64.54(0.04)	64.54(0.04)			
	4%	64.57(0.04)	63.98(0.04)	63.43(0.04)			
	8%	63.43(0.05)	62.30(0.04)	61.26(0.04)			
30%	0%	49.88(0.03)	49.88(0.03)	49.88(0.03)			
	4%	49.73(0.04)	49.48(0.04)	49.24(0.03)			
	8%	48.77(0.04)	48.28(0.04)	47.82(0.03)			
Loaded Participation Rates using (4.4)							
σ_S	σ_r	with $n = 20$			with $n = 100$		
		$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium							
20%	0%	63.82(0.03)	63.82(0.03)	63.82(0.03)	63.99(0.03)	63.99(0.03)	63.99(0.03)
	4%	63.87(0.04)	63.23(0.04)	62.61(0.03)	64.04(0.04)	63.40(0.04)	62.78(0.03)
	8%	62.71(0.05)	61.44(0.04)	60.28(0.03)	62.86(0.05)	61.60(0.04)	60.45(0.03)
30%	0%	49.07(0.03)	49.07(0.03)	49.07(0.03)	49.23(0.03)	49.23(0.03)	49.23(0.03)
	4%	48.95(0.04)	48.66(0.03)	48.37(0.03)	49.11(0.04)	48.82(0.03)	48.54(0.03)
	8%	47.97(0.04)	47.39(0.04)	46.84(0.03)	48.12(0.04)	47.54(0.04)	47.00(0.03)
3% Minimum Guarantee on 90% Premium							
20%	0%	64.37(0.04)	64.37(0.04)	64.37(0.04)	64.47(0.04)	64.47(0.04)	64.47(0.04)
	4%	64.41(0.04)	63.82(0.04)	63.26(0.04)	64.50(0.04)	63.91(0.04)	63.36(0.04)
	8%	63.30(0.05)	62.15(0.04)	61.11(0.03)	63.37(0.05)	62.23(0.04)	61.19(0.03)
30%	0%	49.72(0.03)	49.72(0.03)	49.72(0.03)	49.81(0.03)	49.81(0.03)	49.81(0.03)
	4%	49.59(0.04)	49.33(0.04)	49.09(0.03)	49.67(0.04)	49.42(0.04)	49.18(0.03)
	8%	48.64(0.04)	48.14(0.04)	47.68(0.03)	48.71(0.04)	48.22(0.04)	47.76(0.03)

Table 7: Annual Reset with Term-End Point design (with 20% cap): Impact of correlation, volatilities of index and interest rates on the critical and loaded participation rates

Critical Participation Rates based on (4.2)							
σ_S	σ_r	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$			
3% Minimum Guarantee on 100% Premium							
20%	0%	93.25(0.10)	93.25(0.10)	93.25(0.10)			
	4%	91.20(0.12)	92.00(0.11)	92.84(0.11)			
	8%	86.94(0.13)	88.14(0.13)	89.50(0.12)			
30%	0%	100.46(0.18)	100.46(0.18)	100.46(0.18)			
	4%	95.89(0.21)	97.96(0.21)	100.13(0.20)			
	8%	87.58(0.22)	90.81(0.22)	94.36(0.21)			
3% Minimum Guarantee on 90% Premium							
20%	0%	94.15(0.10)	94.15(0.10)	94.15(0.10)			
	4%	92.02(0.12)	92.98(0.12)	93.99(0.11)			
	8%	87.81(0.14)	89.35(0.13)	91.06(0.12)			
30%	0%	102.44(0.20)	102.44(0.20)	102.44(0.20)			
	4%	97.65(0.22)	99.98(0.22)	102.44(0.22)			
	8%	89.23(0.23)	92.95(0.23)	97.06(0.22)			
Loaded Participation Rates using (4.4)							
σ_S	σ_r	with $n = 20$			with $n = 100$		
		$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium							
20%	0%	92.13(0.09)	92.13(0.09)	92.13(0.09)	92.77(0.10)	92.77(0.10)	92.77(0.10)
	4%	90.19(0.11)	90.91(0.11)	91.67(0.10)	90.76(0.11)	91.53(0.11)	92.34(0.11)
	8%	86.05(0.13)	87.13(0.12)	88.34(0.11)	86.56(0.13)	87.71(0.12)	89.00(0.11)
30%	0%	98.28(0.17)	98.28(0.17)	98.28(0.17)	99.51(0.18)	99.51(0.18)	99.51(0.18)
	4%	93.99(0.20)	95.89(0.20)	97.90(0.19)	95.06(0.20)	97.06(0.20)	99.16(0.19)
	8%	86.08(0.21)	89.06(0.21)	92.33(0.20)	86.93(0.21)	90.05(0.21)	93.48(0.20)
3% Minimum Guarantee on 90% Premium							
20%	0%	93.45(0.10)	93.45(0.10)	93.45(0.10)	93.85(0.10)	93.85(0.10)	93.85(0.10)
	4%	91.39(0.12)	92.30(0.12)	93.26(0.11)	91.75(0.12)	92.69(0.12)	93.67(0.11)
	8%	87.29(0.13)	88.75(0.13)	90.38(0.12)	87.59(0.13)	89.09(0.13)	90.77(0.12)
30%	0%	101.05(0.19)	101.05(0.19)	101.05(0.19)	101.84(0.19)	101.84(0.19)	101.84(0.19)
	4%	96.46(0.21)	98.68(0.21)	101.02(0.21)	97.13(0.22)	99.41(0.22)	101.82(0.21)
	8%	88.35(0.22)	91.90(0.22)	95.82(0.22)	88.85(0.22)	92.50(0.23)	96.52(0.22)

Table 8: Annual Reset with monthly averaging design (without cap): Impact of correlation, volatilities of index and interest rates on the critical and loaded participation rates

Critical Participation Rates based on (4.2)							
σ_S	σ_r	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$			
3% Minimum Guarantee on 100% Premium							
20%	0%	111.79(0.04)	111.79(0.04)	111.79(0.04)			
	4%	111.57(0.05)	110.81(0.04)	110.09(0.04)			
	8%	109.33(0.06)	107.85(0.05)	106.47(0.04)			
30%	0%	84.73(0.03)	84.73(0.03)	84.73(0.03)			
	4%	84.27(0.04)	84.03(0.04)	83.80(0.03)			
	8%	82.37(0.05)	81.89(0.04)	81.43(0.04)			
3% Minimum Guarantee on 90% Premium							
20%	0%	112.53(0.04)	112.53(0.04)	112.53(0.04)			
	4%	112.29(0.05)	111.61(0.05)	110.97(0.04)			
	8%	110.13(0.06)	108.82(0.05)	107.60(0.04)			
30%	0%	85.57(0.04)	85.57(0.04)	85.57(0.04)			
	4%	85.09(0.04)	84.90(0.04)	84.72(0.04)			
	8%	83.24(0.05)	82.87(0.05)	82.53(0.04)			
Loaded Participation Rates using (4.4)							
σ_S	σ_r	with $n = 20$			with $n = 100$		
		$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$	$\rho = -30\%$	$\rho = 0$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium							
20%	0%	111.25(0.04)	111.25(0.04)	111.25(0.04)	111.56(0.04)	111.56(0.04)	111.56(0.04)
	4%	111.06(0.05)	110.29(0.04)	109.55(0.04)	111.35(0.05)	110.59(0.04)	109.86(0.04)
	8%	108.86(0.06)	107.35(0.05)	105.95(0.04)	109.13(0.06)	107.63(0.05)	106.25(0.04)
30%	0%	84.25(0.03)	84.25(0.03)	84.25(0.03)	84.52(0.03)	84.52(0.03)	84.52(0.03)
	4%	83.81(0.04)	83.55(0.04)	83.31(0.03)	84.07(0.04)	83.83(0.04)	83.59(0.03)
	8%	81.93(0.05)	81.43(0.04)	80.96(0.04)	82.18(0.05)	81.69(0.04)	81.23(0.03)
3% Minimum Guarantee on 90% Premium							
20%	0%	112.24(0.04)	112.24(0.04)	112.24(0.04)	112.41(0.04)	112.41(0.04)	112.41(0.04)
	4%	112.01(0.05)	111.33(0.05)	110.68(0.04)	112.17(0.05)	111.49(0.05)	110.84(0.04)
	8%	109.90(0.06)	108.57(0.05)	107.33(0.04)	110.03(0.06)	108.71(0.05)	107.48(0.04)
30%	0%	85.30(0.04)	85.30(0.04)	85.30(0.04)	85.45(0.04)	85.45(0.04)	85.45(0.04)
	4%	84.84(0.04)	84.64(0.04)	84.46(0.03)	84.98(0.04)	84.79(0.04)	84.61(0.03)
	8%	83.02(0.05)	82.64(0.05)	82.28(0.04)	83.14(0.05)	82.77(0.04)	82.42(0.04)

Table 9: Annual Reset with Term-End Point design (100% participation rate): Impact of correlation, volatilities of index and interest rates on the critical cap rates implied from (4.2)

σ_S	σ_r	$\rho = -30\%$	$\rho = 0\%$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium				
20%	0%	19.17(0.01)	19.17(0.01)	19.17(0.01)
	4%	18.87(0.02)	19.01(0.02)	19.14(0.01)
	8%	19.01(0.02)	18.53(0.02)	18.78(0.01)
30%	0%	20.03(0.01)	20.03(0.01)	20.03(0.01)
	4%	19.70(0.02)	19.85(0.02)	20.01(0.01)
	8%	19.01(0.02)	19.31(0.02)	19.60(0.02)
3% Minimum Guarantee on 90% Premium				
20%	0%	19.26(0.01)	19.28(0.01)	19.28(0.01)
	4%	18.97(0.02)	19.13(0.02)	19.28(0.01)
	8%	19.15(0.02)	18.68(0.02)	18.96(0.02)
30%	0%	20.17(0.01)	20.17(0.01)	20.17(0.01)
	4%	19.83(0.02)	20.00(0.02)	20.17(0.01)
	8%	19.15(0.02)	19.47(0.02)	19.79(0.02)

Table 10: Annual Reset with Term-End Point design (100% participation rate and no cap): Impact of correlation, volatilities of index and interest rates on the critical yield spread implied from (4.2)

σ_S	σ_r	$\rho = -30\%$	$\rho = 0\%$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium				
20%	0%	9.11(0.01)	9.11(0.01)	9.11(0.01)
	4%	8.94(0.02)	9.35(0.01)	9.76(0.01)
	8%	9.27(0.02)	10.12(0.02)	10.95(0.02)
30%	0%	21.49(0.03)	21.49(0.03)	21.49(0.03)
	4%	21.27(0.03)	21.81(0.03)	22.36(0.03)
	8%	21.72(0.04)	22.84(0.03)	23.95(0.03)
3% Minimum Guarantee on 90% Premium				
20%	0%	8.80(0.01)	8.80(0.01)	8.80(0.01)
	4%	8.65(0.02)	9.02(0.02)	9.39(0.01)
	8%	8.94(0.02)	9.70(0.02)	10.44(0.02)
30%	0%	20.58(0.03)	20.58(0.03)	20.58(0.03)
	4%	20.37(0.03)	20.89(0.03)	21.34(0.03)
	8%	20.75(0.03)	21.74(0.03)	22.72(0.03)

Table 11: Annual Reset with monthly averaging design (100% participation rate and no cap): Impact of correlation, volatilities of index and interest rates on the critical yield spread implied from (4.2)

σ_S	σ_r	$\rho = -30\%$	$\rho = 0\%$	$\rho = 30\%$
3% Minimum Guarantee on 100% Premium				
20%	0%	-1.439(0.004)	-1.439(0.004)	-1.439(0.004)
	4%	-1.396(0.005)	-1.330(0.005)	-1.264(0.004)
	8%	-1.130(0.007)	-0.987(0.006)	-0.843(0.005)
30%	0%	3.019(0.008)	3.019(0.008)	3.019(0.008)
	4%	3.086(0.009)	3.174(0.009)	3.262(0.008)
	8%	3.471(0.012)	3.661(0.010)	3.847(0.009)
3% Minimum Guarantee on 90% Premium				
20%	0%	-1.505(0.005)	-1.505(0.005)	-1.505(0.005)
	4%	-1.459(0.006)	-1.403(0.005)	-1.348(0.005)
	8%	-1.204(0.007)	-1.085(0.006)	-0.967(0.005)
30%	0%	2.782(0.008)	2.782(0.008)	2.782(0.008)
	4%	2.853(0.010)	2.923(0.009)	2.993(0.008)
	8%	3.211(0.012)	3.362(0.010)	3.509(0.009)