Prior

– $\mu \sim N(\mu_0, \tau_{10}^2)$ ind. of $\nu \sim N(\nu_0, \tau_{20}^2)$

– make change of variable $(\mu, \nu) \to (t\nu, \nu)$
with volume distortion factor $|\nu|$

– joint prior for (t, ν)

$$\frac{|\nu|}{2\pi\tau_{01}\tau_{02}} \exp\left\{-\frac{1}{2}\left[\frac{(t\nu-\mu_0)^2}{\tau_{10}^2} + \frac{(\nu-\nu_0)^2}{\tau_{20}^2}\right]\right\}$$

$$[\;] = \frac{t^2}{\tau_{10}^2}(\nu - \mu_0/t)^2 + \frac{1}{\tau_{20}^2}(\nu-\nu_0)^2$$

use result

$$(\underline{x}-\underline{\mu}_1)' A_1 (\underline{x}-\underline{\mu}_1) + (\underline{x}-\underline{\mu}_2)' A_2 (\underline{x}-\underline{\mu}_2)$$

$$= (\underline{x} - (A_1+A_2)^{-1}(A_1\underline{\mu}_1 + A_2\underline{\mu}_2))'(A_1+A_2)(\;)$$

$$+ (\underline{\mu}_1-\underline{\mu}_2)' A_1 (A_1+A_2)^{-1} A_2 (\underline{\mu}_1-\underline{\mu}_2)$$

with $A_1 = \frac{t^2}{\tau_{10}^2}$, $\mu_1 = \frac{\mu_0}{t}$, $A_2 = \frac{1}{\tau_{02}^2}$, $\mu_2 = \nu_0$ so

$$[\;] = \left(\nu - \left(\frac{t^2}{\tau_{10}^2} + \frac{1}{\tau_{02}^2}\right)^{-1}\left(\frac{t^2}{\tau_{10}^2}\frac{\mu_0}{t} + \frac{\nu_0}{\tau_{20}^2}\right)\right)^2 \left(\frac{t^2}{\tau_{10}^2} + \frac{1}{\tau_{20}^2}\right)$$

$$+ \left(\frac{\mu_0}{t} - \nu_0\right)^2 \frac{t^2}{\tau_{10}^2\tau_{20}^2}\left(\frac{t^2}{\tau_{10}^2} + \frac{1}{\tau_{20}^2}\right)$$

$v_0(t)$

$$= \left(\frac{t^2}{\tau_{10}^2}+\frac{1}{\tau_{20}^2}\right)\left(v-\left(\frac{t^2}{\tau_{10}^2}+\frac{1}{\tau_{02}^2}\right)^{-1}\left(\frac{\mu_0 t}{\tau_{10}^2}+\frac{v_0}{\tau_{20}^2}\right)\right)^2$$

$$+\left(\frac{t^2}{\tau_{10}^2}+\frac{1}{\tau_{20}^2}\right)^{-1}\frac{(\mu_0-v_0 t)^2}{\tau_{10}^2\,\tau_{20}^2}$$

$\tau_{20}^2(t) =$

$$= \tau_{20}^{-2}(t)\,(v-v_0(t))^2+\frac{\tau_{20}^2(t)}{\tau_{10}^2\,\tau_{20}^2}\,(\mu_0-v_0 t)^2$$

∴ prior for t is

$$\frac{1}{2\pi\tau_{01}\tau_{02}}\exp\left\{-\frac{1}{2}\frac{\tau_{20}^2(t)}{\tau_{10}^2\,\tau_{20}^2}(\mu_0-v_0 t)^2\right\}\times$$

$$\int_{-\infty}^{\infty}|v|\exp\left\{-\frac{1}{2\tau_{20}^2(t)}(v-v_0(t))^2\right\}dv$$

$$w = (v-v_0(t))/\tau_{20}(t)$$

$$= \frac{\tau_{20}(t)}{\sqrt{2\pi}\,\tau_{10}\tau_{20}}\exp\left\{-\frac{1}{2}\frac{\tau_{20}^2(t)}{\tau_{10}^2\,\tau_{20}^2}(\mu_0-v_0 t)^2\right\}\times$$

$$\int_{-\infty}^{\infty}|v_0(t)+\tau_{20}(t)w|\,\phi(w)\,dw$$

$$= \frac{\tau_{20}^2(t)}{\sqrt{2\pi}\,\tau_{10}\tau_{20}}\exp\left\{-\frac{1}{2}\frac{\tau_{20}(t)}{\tau_{10}^2\,\tau_{20}^2}(\mu_0-v_0 t)^2\right\}\times$$

$$\int_{-\infty}^{\infty}|w-c(t)|\,\phi(w)\,dw$$

where $c(t) = -v_0(t)/\tau_{20}(t)$

③

$$\int_{-\infty}^{\infty} |w - c(t)|\, \phi(w)\, dw$$

$$= \int_{-\infty}^{c(t)} -(w - c(t))\phi(w)\,dw + \int_{c(t)}^{\infty} (w - c(t))\phi(w)\,dw$$

use $\dfrac{d\,\phi(w)}{dw} = -w\,\phi(w)$

$$= \phi(w)\Big|_{-\infty}^{c(t)} + c(t)\,\Phi(c(t)) - \phi(w)\Big|_{c(t)}^{\infty} - c(t)\,(1 - \Phi(c(t)))$$

$$= 2\,\phi(c(t)) + c(t)\,(2\Phi(c(t)) - 1)$$

$$= 2\,\phi(-c(t)) + 2c(t)\left(1 - \Phi(-c(t)) - \tfrac{1}{2}\right)$$

$$= 2\,\phi(-c(t)) + 2c(t)\left(\tfrac{1}{2} - \Phi(-c(t))\right)$$

$$= 2\left[\phi\left(\frac{v_0(t)}{\tau_{20}(t)}\right) - \frac{v_0(t)}{\tau_{20}(t)}\left(\frac{1}{2} - \Phi\left(\frac{v_0(t)}{\tau_{20}(t)}\right)\right)\right]$$

$\therefore$ prior for t is

$$\frac{2\tau_{20}^2(t)}{\sqrt{2\pi}\,\tau_{10}\tau_{20}} \exp\left\{-\frac{1}{2}\frac{\tau_{20}^2(t)}{\tau_{10}^2\tau_{20}^2}(\mu_0 - v_0 t)^2\right\} \times$$

$$\left\{\phi\left(\frac{v_0(t)}{\tau_{20}(t)}\right) + \frac{v_0(t)}{\tau_{20}(t)}\left(\Phi\left(\frac{v_0(t)}{\tau_{20}(t)}\right) - \frac{1}{2}\right)\right\}$$

– when $\mu_0 = v_0 = 0$ then $v_0(t) = 0$ and the prior is

$$\frac{\tau_{20}^2(t)}{\pi\,\tau_{10}\tau_{20}} = \frac{1}{\pi}\frac{\tau_{20}}{\tau_{10}}\left(1 + \frac{\tau_{20}^2}{\tau_{10}^2}t^2\right)^{-1}$$ which is a scaled Cauchy