

## Solutions - Lecture 2

(1)

1.3.1  $\mathcal{O}$  should have been  $\mathcal{O} = \{\emptyset, \{1, 2, 3, 4, 5\}\}$

Then  $\mathcal{A}(\mathcal{O}) = \{\emptyset, \{1, 2\}, \{3, 4, 5\}, \{3, 4\}, \{1, 2, 5\}, \{1, 2, 3, 4\}, \{5\}, \mathcal{O}\}$

1.3.2  $\{a\} = \bigcap_{n=1}^{\infty} [a - \frac{1}{n}, a]$  and  $[a - \frac{1}{n}, a] \in \mathcal{B}' \forall n$

$[a, b] = \{a\} \cup [a, b]$  and  $\{a\}, [a, b] \in \mathcal{B}'$

$(a, b) = [a, b] \setminus \{a\} = [a, b] \cap \{a\}^c = [a, b] \cap \{a\}^c \cap \{b\}^c$

$[a, b) = [a, b] \setminus \{b\} = [a, b] \cap \{b\}^c$

$(-\infty, b] = \bigcup_{n=1}^{\infty} (b - \frac{1}{n}, b]$

$(a, \infty) = (-\infty, a]^c$

1.3.3  $\{a\} = \bigcap_{n=1}^{\infty} (a - \frac{1}{n}, a]$

$= \bigcap_{n=1}^{\infty} (a - \frac{1}{n}, a] \times \dots \times (a - \frac{1}{n}, a]$

$(a, b) = \bigcup_{n=1}^{\infty} (a, b - \frac{1}{n}]$

$[a, b) = \bigcap_{n=1}^{\infty} \bigcup_{i=1}^{\infty} (a - \frac{1}{n}, b - \frac{1}{i}]$

$= \limsup (a - \frac{1}{n}, b - \frac{1}{n}]$

$[a, b] = \bigcap_{n=1}^{\infty} (a - \frac{1}{n}, b]$ ,  $(-\infty, b] = \bigcup_{n=1}^{\infty} (-n, b]$

1.3.4  $\Sigma_{3/2} (\mu, \Sigma)$

$= \{y : (y - \mu)' \Sigma^{-1} (y - \mu) \leq 9/4\}$

where  $\Sigma = AA' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$   
 $= \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$

so  $\Sigma^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$  so

$= \{y : \begin{pmatrix} y_1 - \mu_1 \\ y_2 - \mu_2 \end{pmatrix}' \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} y_1 - \mu_1 \\ y_2 - \mu_2 \end{pmatrix} \leq 9/4\}$

$= \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} : \begin{pmatrix} (y_1 - \mu_1) - (y_2 - \mu_2) \\ -(y_1 - \mu_1) + 2(y_2 - \mu_2) \end{pmatrix} \begin{pmatrix} y_1 - \mu_1 \\ y_2 - \mu_2 \end{pmatrix} \leq \frac{9}{4} \right\}$

$= \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} : (y_1 - \mu_1)^2 - (y_1 - \mu_1)(y_2 - \mu_2) - (y_1 - \mu_1)(y_2 - \mu_2) + 2(y_2 - \mu_2)^2 \leq \frac{9}{4} \right\}$

$= \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} : (y_1 - \mu_1)^2 - 2(y_1 - \mu_1)(y_2 - \mu_2) + 2(y_2 - \mu_2)^2 \leq \frac{9}{4} \right\}$

and  $\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = Ax_0 + b = A \begin{pmatrix} 0 \\ 0 \end{pmatrix} + b = b = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$= \left\{ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} : (y_1 - 1)^2 - 2(y_1 - 1)(y_2 - 1) + 2(y_2 - 1)^2 \leq \frac{9}{4} \right\}$