Solutions to Exercises - Lecture 8

II.5.1 In Example II.5.3 it is proved that $(X_1, X_2) \sim$ multinomial$(n, p_1, p_2, (1-p_1-p_2))$

So

$$p_{X_1}(x_1) = \sum_{x_2=0}^{n-x_1} p_{(X_1,X_2,n-X_1-X_2)}(x_1,x_2,n-x_1-x_2)$$

$$= \sum_{x_2=0}^{n-x_1} \binom{n}{x_1\ x_2\ n-x_1-x_2} p_1^{x_1} p_2^{x_2} (1-p_1-p_2)^{n-x_1-x_2}$$

$$= \sum_{x_2=0}^{n-x_1} \frac{n!}{x_1!\, x_2!\, (n-x_1-x_2)!} p_1^{x_1} p_2^{x_2} (1-p_1-p_2)^{n-x_1-x_2}$$

$$= \frac{n!}{x_1!\,(n-x_1)!} p_1^{x_1} (1-p_1)^{n-x_1} \times \sum_{x_2=0}^{n-x_1} \frac{(n-x_1)!}{x_2!\,(n-x_1-x_2)!} \left(\frac{p_2}{1-p_1}\right)^{x_2} \left(1-\frac{p_2}{1-p_1}\right)^{n-x_1-x_2}$$

" sum of all binomial$\left(n-x_1, \frac{p_2}{1-p_1}\right)$ probabilities and so equals 1

$$= \binom{n}{x_1} p_1^{x_1} (1-p_1)^{n-x_1}$$ and so $X_1 \sim$ binomial(n, p_1)

II.5.2 Note $Y = X_1 + \cdots + X_l$ equals the number of responses falling in the first l categories and a response falls into one of these categories with prob. $p_1 + \cdots + p_l$. Therefore $Y \sim$ binomial$(n, p_1 + \cdots + p_l)$.

II.5.3 If $A \in \mathcal{A}$ then $I_A : \Omega \to R^1$

and $I_A^{-1} B = \{\omega : I_A(\omega) \in B\}$

$$= \begin{cases} \Omega & \text{if } 0, 1 \in B \\ A^c & \text{if } 0 \in B,\ 1 \notin B \\ A & \text{if } 0 \notin B,\ 1 \in B \\ \phi & \text{if } 0 \notin B,\ 1 \notin B \end{cases}$$

and so $I_A^{-1} B$. Since this is true $\forall B \in \mathcal{B}^1$ we have that $Y = I_A$ is a r.v. which takes values in $\{0, 1\}$. Then

$$P_Y(1) = P(I_A^{-1}\{1\}) = P(\{\omega : I_A(\omega) = 1\})$$

$$= P(A)$$

Therefore $Y = I_A \sim \text{Bernoulli}(P(A))$