Week 10: Three factor experiments. The GLM and missing observations. Randomized block design. Factorial experiments run in complete blocks. Latin square design. [pp153-168]

Three factor Anova

- interpretations are similar except we are averaging over the factors unmentioned in the expression p 154 CN

Example p155 CN

Results for: POTATO.MTW

Data Display

Row	Bacteria	Temp	Oxygen	Rot
1	1	1	1	7
2	1	1	1	7
3	1	1	1	9
4	1	1	2	0
5	1	1	2	0
6	1	1	2	0
7	1	1	3	9
8	1	1	3	0
9	1	1	3	0
10	1	2	1	10
11	1	2	1	6
12	1	2	1	10
13	1	2	2	4
14	1	2	2	10
15	1	2	2	5
16	1	2	3	8
17	1	2	3	0
18	1	2	3	10
19	2	1	1	2
20	2	1	1	4
21	2	1	1	9
22	2	1	2	4
23	2	1	2	5
24	2	1	2	10
25	2	1	3	4
26	2	1	3	5
27	2	1	3	0
28	2	2	1	17
29	2	2	1	18

30	2	2	1	8
31	2	2	2	3
32	2	2	2	23
33	2	2	2	7
34	2	2	3	15
35	2	2	3	14
36	2	2	3	17
37	3	1	1	13
38	3	1	1	11
39	3	1	1	3
40	3	1	2	10
41	3	1	2	4
42	3	1	2	7
43	3	1	3	15
44	3	1	3	2
45	3	1	3	7
46	3	2	1	26
47	3	2	1	19
48	3	2	1	24
49	3	2	2	15
50	3	2	2	22
51	3	2	2	18
52	3	2	3	20
53	3	2	3	24
54	3	2	3	8

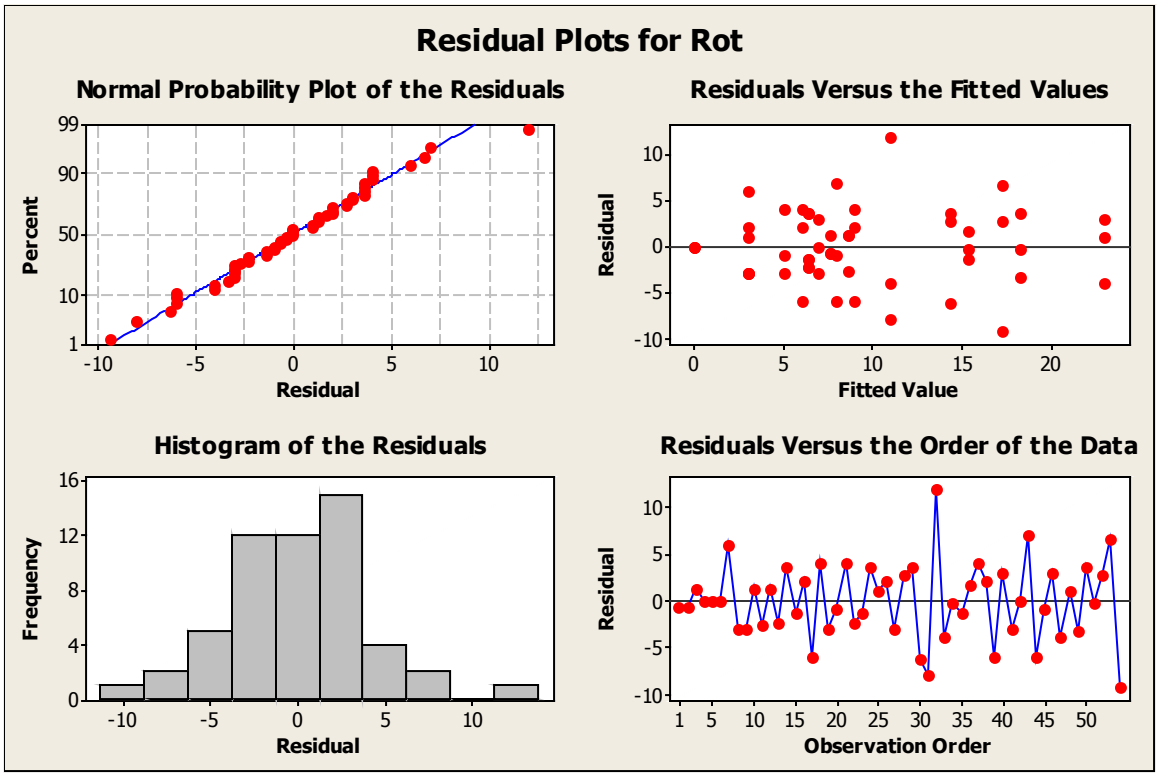
ANOVA: Rot versus Bacteria, Temp, Oxygen

Factor	Type	Levels	Values
Bacteria	fixed	3	1, 2, 3
Temp	fixed	2	1, 2
Oxygen	fixed	3	1, 2, 3

Analysis of Variance for Rot

Source	DF	SS	MS	F	P
Bacteria	2	651.81	325.91	13.91	0.000
Temp	1	848.07	848.07	36.20	0.000
Oxygen	2	97.81	48.91	2.09	0.139
Bacteria*Temp	2	152.93	76.46	3.26	0.050
Bacteria*Oxygen	4	30.07	7.52	0.32	0.862
Temp*Oxygen	2	1.59	0.80	0.03	0.967
Bacteria*Temp*Oxygen	4	81.41	20.35	0.87	0.492
Error	36	843.33	23.43		
Total	53	2707.04			

S = 4.84003 R-Sq = 68.85% R-Sq(adj) = 54.14%



Descriptive Statistics: Rot

Results for Bacteria = 1

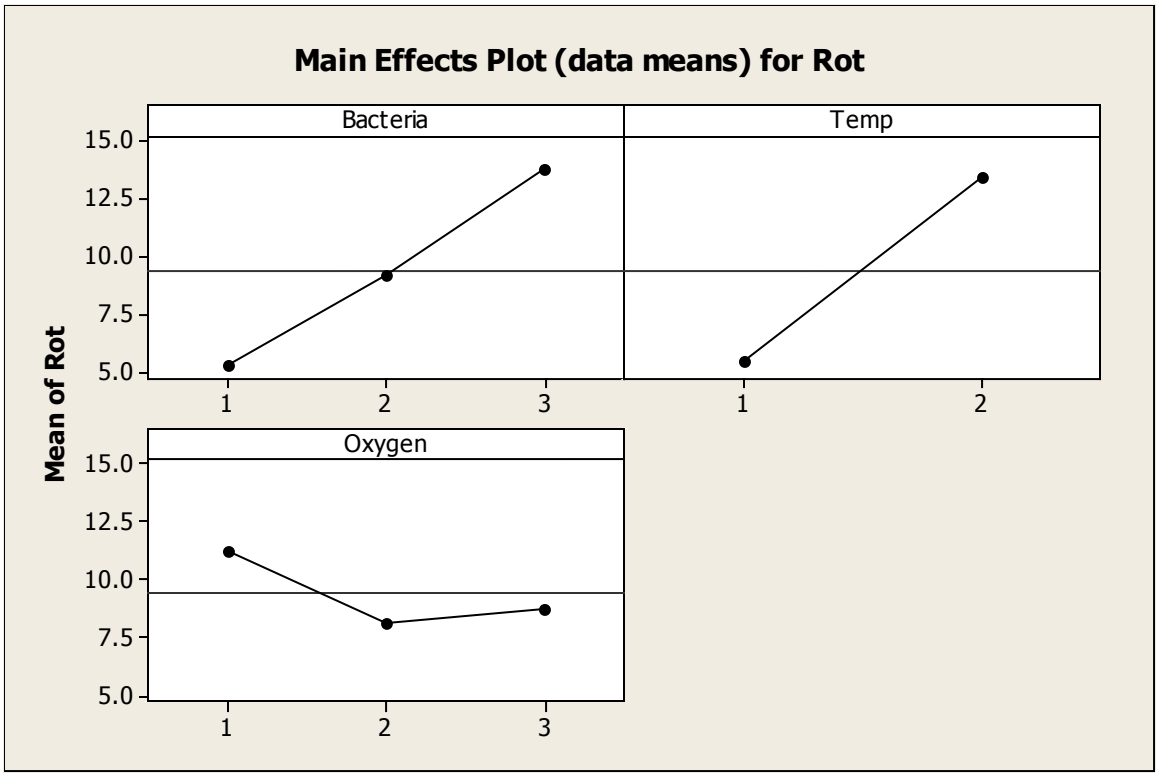
Variable	Temp	N	Mean	StDev
Rot	1	9	3.56	4.28
	2	9	7.00	3.54

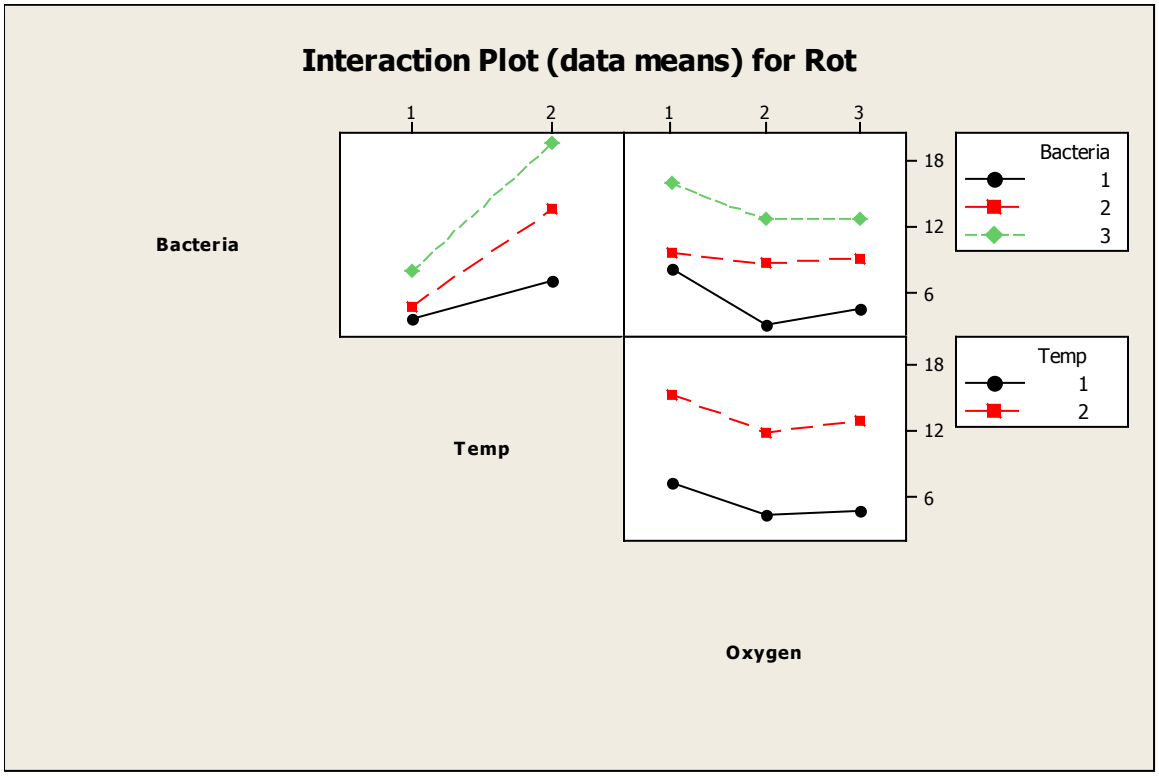
Results for Bacteria = 2

Variable	Temp	N	Mean	StDev
Rot	1	9	4.78	3.11
	2	9	13.56	6.33

Results for Bacteria = 3

Variable	Temp	N	Mean	StDev
Rot	1	9	8.00	4.56
	2	9	19.56	5.5


Main Effects Plot (data means) for Rot
Bacteria
Temp
Oxygen
Mean of Rot
15.0
12.5
10.0
7.5
5.0
1
2
3


Interaction Plot (data means) for Rot
Bacteria
Temp
Oxygen
Bacteria
1
2
3
Temp
1
2
18
12
6

The case when n=1 (one observation per cell) p 158 CN

- error d.f. =0

- - if we can assume that the interaction is negligible, we can use the interaction MS to test the main effects

- The model $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$ (with no interaction term)

-ANOVA table with two factors A (with a levels) and B (with b levels)

Randomized block design (RBD) p159 CN

- Model $y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}$
- A noise reducing design

Example 4 p161 CN

Tabulated statistics: Block, Method

Rows: Block Columns: Method

	M1	M2	M3	M4
B1	7	5	3	3
B2	9	9	5	4
B3	4	6	2	3

Data Display

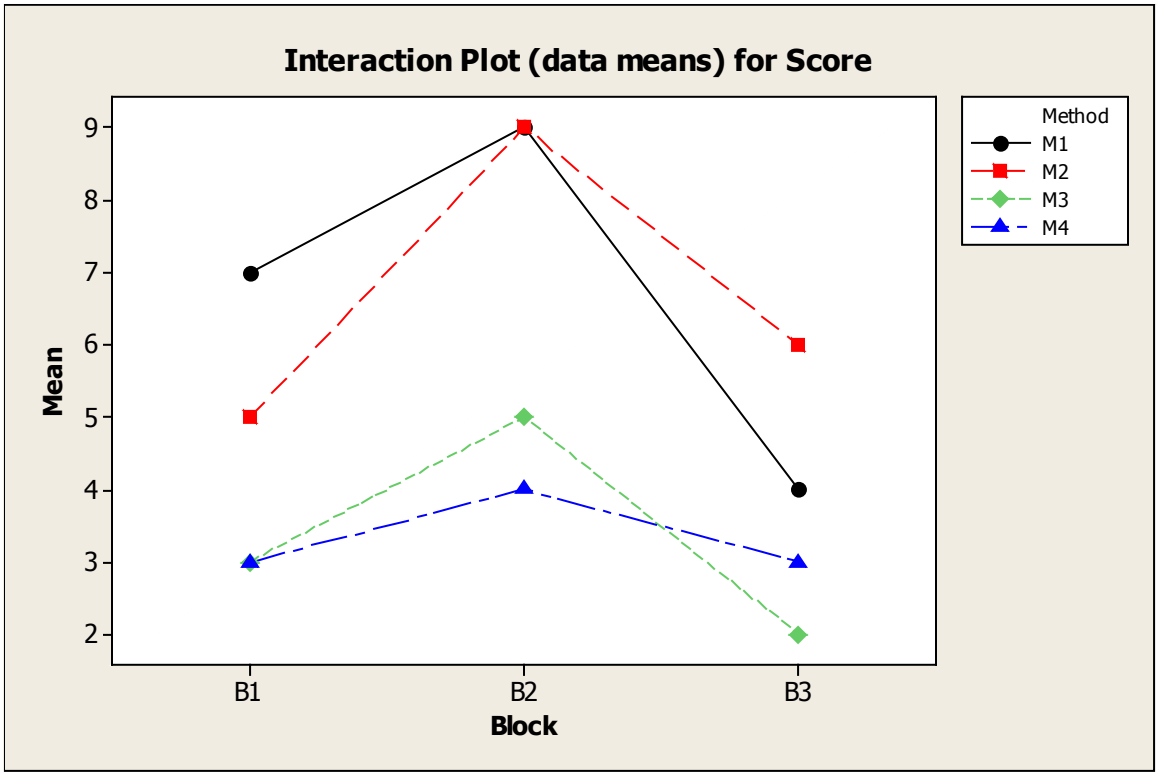
Row	Block	Method	Score
1	B1	M1	7
2	B1	M2	5
3	B1	M3	3
4	B1	M4	3
5	B2	M1	9
6	B2	M2	9
7	B2	M3	5
8	B2	M4	4
9	B3	M1	4
10	B3	M2	6
11	B3	M3	2
12	B3	M4	3

Two-way ANOVA: Score versus Method, Block

Source	DF	SS	MS	F	P
Method	3	33.3333	11.1111	9.30	0.011
Block	2	19.5000	9.7500	8.16	0.019
Error	6	7.1667	1.1944		
Total	11	60.0000			

S = 1.093 R-Sq = 88.06% R-Sq(adj) = 78.10%

Note: We are assuming no treatment block interaction and so we should look at a plot of the data to see if this assumption is reasonable



Confidence intervals for the differences for the differences between treatment means

Treatment means: $\bar{T}_i - \bar{T}_j \pm t_{\alpha/2} s\sqrt{\frac{2}{b}}$

$s = \sqrt{MSE}$

- Use Tukey's method if you want all pairwise CIs

Ex Find a 95% CI for the difference between mean scores of methods 1 and 2.
Here is the data:

Rows: Block Columns: Method

	M1	M2	M3	M4
B1	7	5	3	3
B2	9	9	5	4
B3	4	6	2	3

- If some observations are missing, the procedure doesn't work. Use GLM procedure (with indicator variables)

Ex In a manufacturing process, a plastic rod is produced by heating a granular plastic to a molten state and then extruding it under pressure through a nozzle. An experiment was conducted to investigate the effect of two factors, extrusion temperature (^{o}F) and pressure (pounds per square inch). A complete 2 x 2 factorial experiment was conducted. Three batches of granular plastic were used in the experiment, with each batch (viewed as a block) divided into four equal parts. The four portions of granular plastic of a given batch were randomly assigned to the four treatments; this was repeated for each of the three batches, resulting a 2 x 2 factorial experiment laid out in three blocks. The data are shown below:

Results for: RODMOLD.MTW

Data Display

Row	Batch(Block)	Temperature	Pressure	Rate of Extrusion
1	1	200	40	1.35
2	1	200	60	1.74
3	1	300	40	2.48
4	1	300	60	3.63
5	2	200	40	1.31
6	2	200	60	1.67
7	2	300	40	2.29
8	2	300	60	3.30
9	3	200	40	1.40
10	3	200	60	1.86
11	3	300	40	2.14
12	3	300	60	3.27

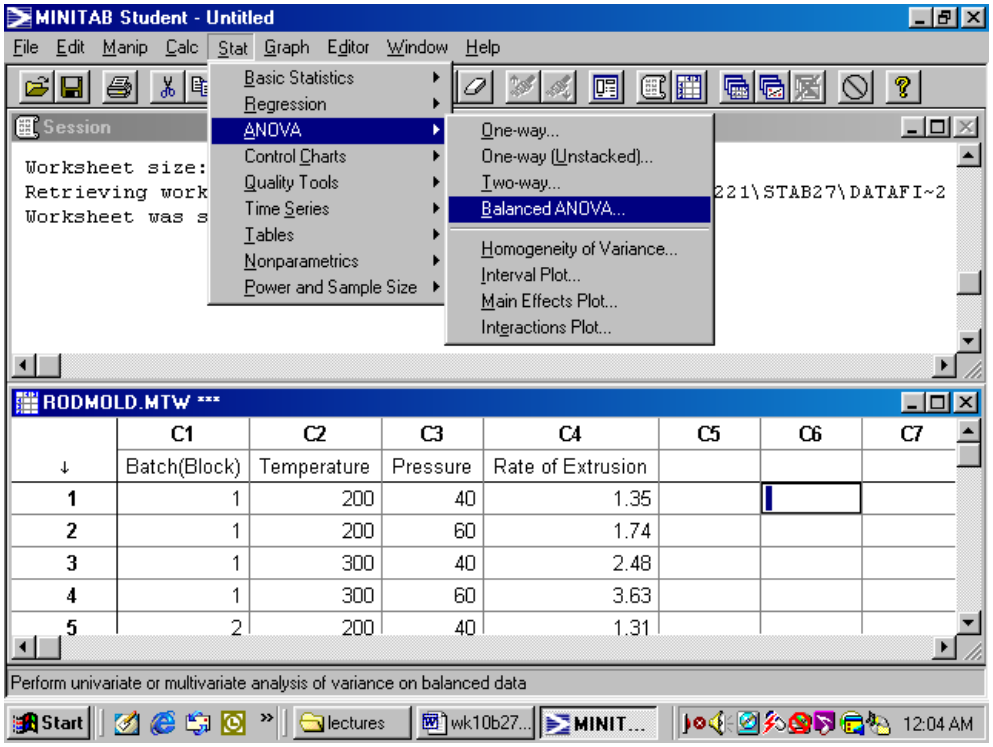
ANOVA: Rate of Extrusion versus Treatment, Batch(Block)

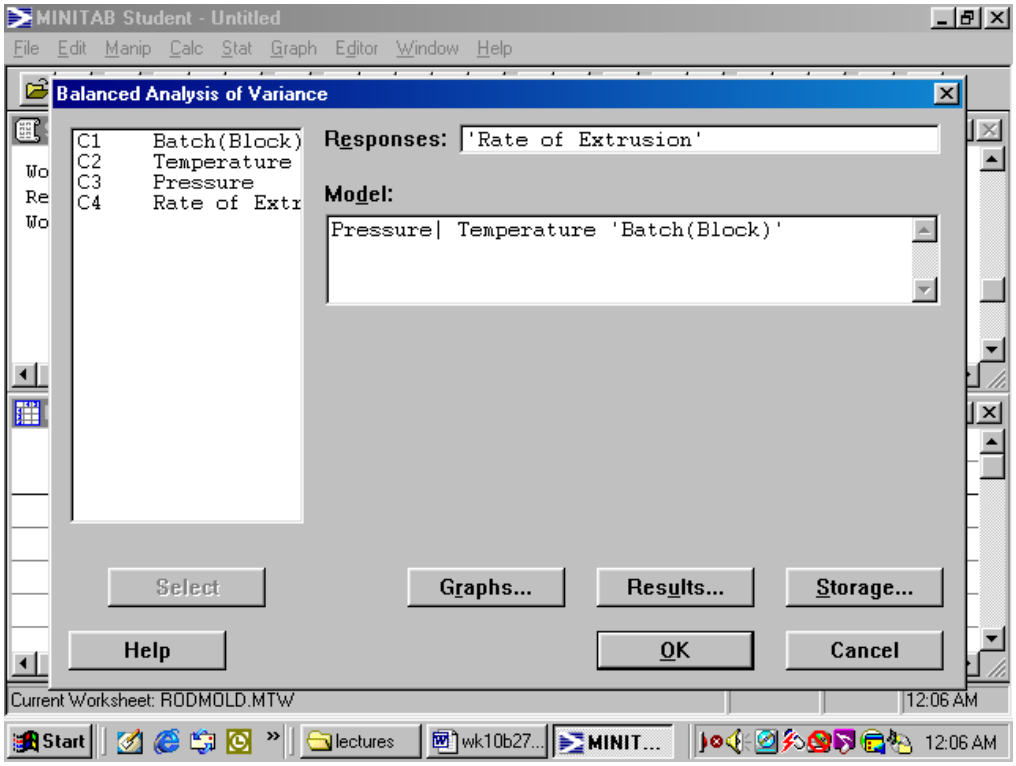
```
Factor        Type   Levels  Values
Treatment     fixed       4  20040, 20060, 30040, 30060
Batch(Block)  fixed       3  1, 2, 3


Analysis of Variance for Rate of Extrusion

Source        DF      SS      MS       F      P
Treatment      3  7.0921  2.3640  137.60  0.000
Batch(Block)   2  0.0573  0.0287    1.67  0.265
Error          6  0.1031  0.0172
Total         11  7.2525


S = 0.131075   R-Sq = 98.58%   R-Sq(adj) = 97.39%
```

ANOVA: Rate of Extrusion versus Temperature, Pressure, Batch(Block)

```
Factor        Type   Levels  Values
Temperature   fixed       2  200, 300
Pressure      fixed       2  40, 60
Batch(Block)  fixed       3  1, 2, 3


Analysis of Variance for Rate of Extrusion

Source                DF      SS      MS       F      P
Temperature            1  5.0440  5.0440  293.59  0.000
Pressure               1  1.6875  1.6875   98.22  0.000
Temperature*Pressure   1  0.3605  0.3605   20.98  0.004
Batch(Block)           2  0.0573  0.0287    1.67  0.265
Error                  6  0.1031  0.0172
Total                 11  7.2525


S = 0.131075   R-Sq = 98.58%   R-Sq(adj) = 97.39%
```

MIMITAB Commands





Latin Square Design p166 CN

An experiment was conducted to study the effects of different types of background music on the productivity of bank employees. The treatments were defined as various combinations of tempo music.

Treatments:
A : Slow instrumental and vocal
B: Medium, instrumental and vocal
C: Fast, instrumental and vocal
D: medium, instrumental only
E: fast, instrumental only

The data from the experiment are shown below (the treatment in each cell is shown in parenthesis)

Week	Day				
	M	T	W	Th	F
1	18 (D)	17 (C)	14 (A)	21 (B)	17 (E)
2	13 (C)	34 (B)	21 (E)	16 (A)	15 (D)
3	7 (A)	29 (D)	32 (B)	27 (B)	13 (C)
4	17 (E)	13 (A)	24 (C)	31 (D)	25 (B)
5	21 (B)	26 (E)	26 (D)	31 (C)	7 (A)

- Model $y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \varepsilon_{ijk}$

- A noise reducing design

Results for: Latinsq_eg_p1186Neter.MTW

Two-way ANOVA: Productivity versus Week, Day

Source	DF	SS	MS	F	P
Week	4	82.0	20.5	0.38	0.816
Day	4	477.2	119.3	2.24	0.111
Error	16	852.8	53.3		
Total	24	1412.0			

S = 7.301 R-Sq = 39.60% R-Sq(adj) = 9.41%

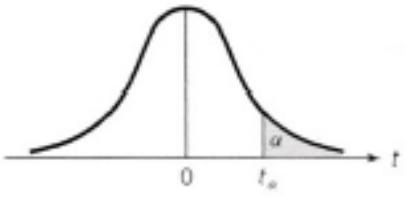
One-way ANOVA: Productivity versus Treatment

Source	DF	SS	MS	F	P
Treatment	4	664.4	166.1	4.44	0.010
Error	20	747.6	37.4		
Total	24	1412.0			

(The anova table below is not from MINITAB)

Source	DF	SS	MS	F	P
Treatment	4	664.4	166.1	10.57	
Week	4	82.0	20.5	1.31	
Day	4	477.2	119.3	7.60	
Error	12	188.4	15.7		
Total	24	1412.0			

TABLE C.2 **Critical Values for Student's *t***



ν	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.001}$	$t_{.0005}$
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291