

STA 3000, Spring 2014, Assignment #2. Due in class on March 3.

*These questions are to be answered by each student individually. Any discussions you have with other people about these questions should concern general issues only, and should not result in your taking away written or electronically-recorded notes.*

**Question 1:** Consider a decision problem in which the model parameter,  $\theta$ , is any integer, the distribution for the integer observation,  $y$ , given  $\theta$  is

$$P(Y = y | \theta) = \begin{cases} 1/3 & \text{if } y \in \{\theta - 1, \theta, \theta + 1\} \\ 0 & \text{otherwise} \end{cases}$$

the action space is the integers, and we use 0-1 loss:

$$L(\theta, a) = \begin{cases} 0 & \text{if } a = \theta \\ 1 & \text{if } a \neq \theta \end{cases}$$

Define the decision rule  $\delta$  by  $\delta(y) = y$ . Find decision rules  $\delta'$  and  $\delta''$  such that  $\delta'$  dominates  $\delta$ ,  $\delta''$  dominates  $\delta'$ , and  $\delta''$  is admissible (and prove that it is admissible).

**Question 2:** Consider a decision problem in which the model parameter,  $\theta$ , is any real, the distribution for the real observation,  $y$ , given  $\theta$ , is uniform over the interval  $(\theta - 1/2, \theta + 1/2)$ , the action space is the reals, and we use the following loss function:

$$L(\theta, a) = \min(|\theta - a|, 1/6)$$

Show that the decision rule  $\delta(y) = y$  is inadmissible.

**Question 3:** Consider  $K$  decision problems, in which the the  $k$ 'th decision problem has data  $y_k$ , modelled using parameter  $\theta_k$ , with action  $a_k$  resulting in loss  $L_k(\theta_k, a_k)$ . Suppose that for the  $k$ 'th decision problem we use the prior  $P(\theta_k)$ , and that the decision rule  $\delta_k$  minimizes posterior expected loss with this prior.

Now consider a combined decision problem, with data  $y = (y_1, \dots, y_K)$ , modelled using parameter  $\theta = (\theta_1, \dots, \theta_K)$ , as follows:

$$P(y|\theta) = \prod_{k=1}^K P(y_k|\theta_k)$$

Let the action space for this decision problem be the Cartesian product of the action spaces for the  $K$  original problems, and let the loss function be

$$L(\theta, a) = \sum_{k=1}^K L_k(\theta_k, a_k)$$

Prove that the decision rule

$$\delta(y) = (\delta_1(y_1), \dots, \delta_K(y_K))$$

minimizes posterior expected loss if the prior is

$$P(\theta) = \prod_{k=1}^K P(\theta_k)$$

where  $P(\theta_k)$  is the prior for the  $k$ 'th original decision problem.